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# Multi-attribute decision-making methods based on normal random variables in supply chain risk management

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## Abstract

Random multi-attribute decision-making is a finite option selection problem related to multiple attributes, and the attribute values are random variables. Its application and supply chain risk management can transform interval decision numbers and fuzzy decision numbers into standardised decisions. Based on this research background, the article provides a primary method to determine the randomness of standard random variables based on expectations and variance through theoretical analysis. Second, the article determines the range of the total utility value of each supply chain selection plan based on the $3\sigma$ principle. Experiments have proved that this method can solve unifying opinions due to different knowledge, experience, and preferences of evaluation experts. This provides a new method of supplier selection.

**Keywords:** normal random variables, mixed multi-attribute decision-making, entropy, subjective weight, objective weight, supply chain risk management
**AMS 2010 codes:** 90B50

## 1 Introduction

The complexity of decision-making issues leads to decision-making indicators, often including quantitative and qualitative indicators. The hybrid multi-attribute decision-making model can handle quantitative and qualitative indicators, which is more in line with actual decision-making situations. However, due to the complexity of the attributes and the bounded rationality of the decision-maker, it is difficult for the weight directly given by the decision maker's subjective judgement to be consistent with the actual situation [1]. The article presents a mathematical programming model that integrates decision-makers personal weight preference information and objective decision matrix information. At the same time, we propose a combined weight algorithm that can integrate all kinds of subjective weights and n-1 kinds of objective weights.

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## 2 Mixed multi-attribute decision-making problems

Suppose $S = \{s_1, s_2, \cdots, s_m\}$ is a set of solutions for a multi-attribute decision-making problem. $U = \{u_1, u_2, \cdots, s_n\}$ is the indicator set. The weight vector of the indicator is $W = \{w_1, w_2, \cdots, w_n\}$ which is unknown [2].

**Definition 1.** We define $a = [a^L, a^U]$ as a closed interval number. Among them, $a^L$, $a^U \in R$ and the total number of intervals on $0 \leq a^L \leq a^U$, $R$ are denoted as $R$.

**Definition 2.** We assume that $[a^L, a^U]$ is an interval number. $\rho$: $[0,1] \to [0,1]$ is a function with the following properties: $\rho$: $(0) - 0$; $\rho(1) = 1$; if $x \geq \gamma$ then $\rho(x) \geq \rho(\gamma)$, and

$$f_\rho\left([a^L, a^U]\right) = \int_0^1 \frac{d\rho(\gamma)}{d\gamma}\left(a^U - \gamma(a^U - a^L)\right) d\gamma \tag{1}$$

$$f_\gamma(a^L, a^U]) = (a^L + a^U])/2 \tag{2}$$

**Definition 3.** We assume that $R$ is a set of real numbers. $P(R)$ represents the set of all fuzzy subsets on $R$. A fuzzy set $\widetilde{A} \in P(R)$ is called a fuzzy number. If there is at least one $x_0 \in R$, even $u_A(x_0) = 1$, $\widetilde{A}$ is standard [3].

**Definition 4.** The fuzzy maximum set is a fuzzy subset $S_{\max} = \{(x, u_{\max})\,|x \in R\}$, and its membership function is:

$$\mu_{\min}(x) = \begin{cases} 1 - x, & 0 \leq x \leq 1 \\ 0 & otherwise \end{cases} \tag{3}$$

In this way, the fuzzy number $\widetilde{A}$ can be converted into the exact number $b$:

$$\begin{aligned} b &= [\mu_R A + 1 - \mu_L(A)]/2 \\ \mu_R(A) &= \sup[\mu_A(x)\Lambda\mu_{\max}(x)] \\ \mu_L(A) &= \sup[\mu_A(x)\Lambda\mu_{\min}(x)] \end{aligned} \tag{4}$$

In this way, the mixed decision matrix $A = (a_{ij})_{m\times n}$ is transformed into an exact number matrix $B = (b_{ij})_{m\times n}$ through Eqs (2) and (4).

$$c_{ij} = b_{ij}/\sqrt{\sum_{i=1}^{m} b_{ij}^2} \tag{5}$$

The positive ideal solution is $A^* = \left\{c_1{}^*, \cdots c_j{}^* \cdots, c_n{}^*\right\}$, where $c_j{}^* = \{\max_i c_{ij}, j \in J_1; \min_i c_{ij}, j \in J_2$; the negative ideal solution is $\overline{A} = \left\{\overline{c}_1, \cdots, \overline{c}_j \cdots, \overline{c}_n\right\}$. $J_1$ is a profitable attribute index. $J_2$ is the cost attribute index.

## 3 Algorithms for comprehensive weights

### 3.1 Insufficiency of the existing objective weight calculation model

After studying various methods of determining objective weights, some scholars have proposed mathematical optimisation models [4]. These models often use the following methods when solving objective weights. We transform the exact number decision matrix $A = (a_{ij})_{m\times n}$ into a standardised decision matrix $B = (b_{ij})_{m\times n}$.

$$b_{ij} = \frac{a_{ij} - a_j^{\min}}{a_j^{\max} - a_j^{\min}}, \quad j \in J_1; \tag{6}$$

$$b_{ij} = \frac{a_j^{\max} - a_{ij}}{a_j^{\max} - a_j^{\min}}, \quad j \in J_2; \tag{7}$$

$J_1$ is a profit-based indicator. $J_2$ is a cost index. In this way, a solution model for objective weights is obtained (8)

$$\begin{cases} \min Z_1 = \sum\limits_{j=1}^{n} w^T H w \\ s.t. e^T w = 1, \quad w_j \geq 0 \end{cases} \tag{8}$$

$H$ is the diagonal matrix of $n \times n$. Its diagonal element is $h_{ij} = \sum\limits_{i=1}^{m} (b_{ij} - b_j{}^*)^2$, $b_j{}^* = \max \{b_{1j} \cdots b_{mj}\}$. Solving model (8) can get:

$$w = H^{-1} e / e^T H^{-1} e \tag{9}$$

Some scholars pointed out that the weight distribution mechanism as well as the meaning of model (8) is not precise, and it does not conform to the principle of entropy model weight distribution. Through case analysis, it is found that small changes in the decision matrix will lead to significant changes in weights, so the weight distribution mechanism of the model (8) is unreasonable. So, we proposed an entropy model to solve the objective weights [5]. The main methods are as follows:

$$w_j = d_j / \sum_{j=1}^{n} d_j \tag{10}$$

$$d_j = 1 - E_j, \quad E_j = -\left( \sum_{i=1}^{n} p_{ij} \ln p_{ij} \right) / \ln n p_{ij} = a_{ij} / \sum_{i=1}^{m} a_{ij}$$

To assign weights, the entropy model is guided by the following principle. If the evaluation value of each scheme under the $j$ attribute tends to be more consistent, then the weight of the $j$ attribute will be smaller. The entropy model also has some unreasonable points in assigning weights as follows:

1. The weight distribution is not flexible. The entropy model defines $d_j = 1 - E_j$, so can $d_j = 2 - E_j$ or other functions of $E_j$ being set?
2. It is easy to cause too much weight difference. In actual decision-making, when an indicator is introduced into the evaluation system, it can generally be considered that it cannot exceed and equal to zero [6]. That is, the maximum weight cannot be $>$10 times the minimum weight.

The construction of a suitable mathematical model requires a deep understanding of the specific situation and rich mathematical experience of the issues involved in the decision-making problem. This isn't easy. To judge the rationality of the objective weight model, we give Judgement Theorem 1.

**Judgement Theorem 1.** The objective weight obtained by this model can reflect the information of the decision matrix. When the decision matrix changes, the degree of weight change should be consistent with the degree of change of the decision matrix.

### 3.2 Entropy coefficient model

Based on the model (8) and entropy model (10), we transform the exact number decision matrix $A = (a_{ij})_{m \times n}$ into a standardised decision matrix $C = (c_{ij})_{m \times n}$. Among them:

$$c_{ij} = \frac{a_{ij}}{a_j^{\max}}, \quad j \in J_1; \tag{11}$$

$$c_{ij} = \frac{a_j^{\min}}{a_{ij}}, \quad j \in J_2; \tag{12}$$

$J_1$ is a profit-based indicator and $J_2$ is a cost index;

$$a_j^{\max} = \max\left\{a_{1j}, a_{2j}, \cdots, a_{mj}\right\}, \quad j = 1, \cdots, n; \qquad (13)$$
$$a_j^{\min} = \min\left\{a_{1j}, a_{2j}, \cdots, a_{mj}\right\}, \quad j = 1, \cdots, n;$$

**Definition 5.** For the normalised matrix $C = (c_{ij})_{m\times n}$, the entropy of the $j$ attribute is defined as:

$$h_j = \rho - E_j \qquad (14)$$

$E_j = -\left(\sum_{i=1}^{n} c_{ij} \ln c_{ij}\right) / \ln n$, $\rho$ is the system parameter $(\rho \geq \max\{E_1, \cdots E_j \cdots E_n\})$. Then the entropy coefficient model for solving the objective weight is:

$$\begin{cases} \min Z_2 = w^T K w \\ s.t. e^T w = 1 \\ w \geq 0 \end{cases} \qquad (15)$$

where $K$ is the diagonal matrix of $n \times n$. Its diagonal elements are $k_{ij} = \rho - E_j, k_{ij} > 0, j = 1, \cdots, n$; the remaining elements are zero. We assume that $L = w^T K w - \lambda\left(e^T w - 1\right)$, then $\frac{\partial L}{\partial w_j} = 2Kw - \lambda = 0$, $\frac{\partial L}{\partial \lambda} = e^T w - 1 = 0$. Calculate to get $w = K^{-1}e/e^T K^{-1}e$.

**Property 1.** The weight distribution principle of the entropy coefficient model is the same as that of the entropy model. If the evaluation value of each scheme under the jth attribute tends to be more consistent, then the weight of the jth attribute will be smaller.

**Property 2.** The entropy coefficient model has certain flexibility [7]. The decision-maker can set the size of the system parameter $\rho$ according to specific actual needs to adjust the degree of the weight difference between attributes. The larger the $\rho$, the smaller the system attribute weight difference.

### 3.3 Comparison between models (8), (10), and (14)

Here are two examples to illustrate the difference between the entropy coefficient model (14), the model (8), and the entropy model (10):

**Example 1.** Suppose there is a decision matrix $A_{m\times n} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$. We use model (8) to normalise $A$ to get matrix $B$, then $w_j = \frac{(h_{jj})^{-1}}{\sum_{j=1}^{n} (h_{jj})^{-1}}$ can be obtained according to formula (9). If the evaluation value of each scheme under the $j$ attribute tends to be the same [8]. That is, $b_{ij} \to b_j{}^* \,(i = 1, 2, \cdots, m)$ is $h_{ij} \to 0$, so $w_j = \lim_{h_{jj}\to 0} \frac{(h_{jj})^{-1}}{\sum_{j=1}^{n} (h_{jj})^{-1}} = 1$. Model (8) may cause the weight of the $j$ attribute to be too large.

We use the entropy model (10) to calculate. If the evaluation value of each scheme under the jth attribute tends to be the same, that is, $p_{ij} \to 1/n \,(i = 1, 2 \cdots m)$ is $d_j \to 0$, so $w_j = \frac{d_j}{\sum_{j=1}^{n} d_j} \to 0$. When assigning weights, the degree of weight difference may be too large.

3. We use the entropy coefficient model (15) for solving objective weight. According to formula (15), we can get: $w_j = \frac{(k_{jj})^{-1}}{\sum_{j=1}^{n} (k_{jj})^{-1}}$. If the evaluation value of each scheme under the jth attribute tends to be the same, that is, $c_{ij} \to 1 \,(i = 1, 2, \cdots, m)$ is $E_j \to 0$. So $k_{jj} = \rho - E_j \to \rho$, then

$$w_j = \lim_{k_{jj} \to \rho} \frac{(k_{jj})^{-1}}{\sum\limits_{j=1}^{n} (k_{jj})^{-1}} = \frac{\rho^{-1}}{\sum\limits_{i \neq j}^{n} (k_{jj})^{-1} + \rho^{-1}} \tag{16}$$

In this way, we can set the system parameters $\rho$ according to the specific decision-making situation so that the entropy coefficient model (15) has a certain degree of flexibility [9].

There is a decision matrix $A_{4\times4}$. To simplify, we assume that its attribute indicators are all income indicators

$$A = \begin{bmatrix} & P_1 & P_2 & P_3 & P_4 \\ S_1 & 30 & 30 & 38 & 29.0 \\ S_2 & 19 & 54 & 86 & 29.0 \\ S_3 & 19 & 15 & 85 & 28.9 \\ S_4 & 68 & 70 & 60 & 29.0 \end{bmatrix} \tag{17}$$

1. Using model (8), we can get: w = (0.1384, 0.2232, 0.2783, 0.3601).
2. Using the entropy model (10), we can get: w = (0.4630, 0.3992, 0.1378, 0).
3. Using the entropy coefficient model (14), when the system parameter is $\rho = 0.8$, we can get w = (0.7875, 0.1296, 0.0576, 0.0253). When the system parameters are used, $\rho = 1$ can get w = (0.4404, 0.2795, 0.1806, 0.0996). When the element $a_{34}$ of the matrix $A$ changes from 28.9 to 29.1, we can get the matrix $A_1$:

$$A_1 = \begin{bmatrix} & P_1 & P_2 & P_3 & P_4 \\ S_1 & 30 & 30 & 38 & 29.0 \\ S_2 & 19 & 54 & 86 & 29.0 \\ S_3 & 19 & 15 & 85 & 28.9 \\ S_4 & 68 & 70 & 60 & 29.0 \end{bmatrix} \tag{18}$$

1. Using model (8), we can get w=(0.1821, 0.2937, 0.3662, 0.1579).
2. Using the entropy model (10), we can get w=(0.4630, 0.3992, 0.1378, 0).
3. Use the entropy coefficient model (14). When the system parameter is $\rho = 0.8$, w = (0.7874, 0.1296, 0.0576, 0.0254) can be obtained. When the system parameters are used, $\rho = 1$ can be w = (0.4402, 0.2793, 0.1805, 0.1).

When $a_{34}$ undergoing a small change, the weight change obtained using model (8) is too large [10]. The weight of the particular attribute P4 has changed from 0.3601 to 0.1579. The weights obtained by using the entropy model (10) have not changed. This cannot reflect a slight change in the decision matrix. Using the entropy coefficient model (14), the weight change obtained is relatively small, consistent with the slight chance of the matrix. From the aforementioned two examples, the entropy coefficient model can adapt to different decision-making situations by adjusting the value of the system parameter $\rho$, and the weight obtained is more reasonable than the model (8) and the entropy model (10).

### 3.4 Comprehensive weight calculation method

Suppose that the decision-maker directly gives the emotional weight of the attribute as $W^{(0)} = \left(w_1^{(0)}, \cdots, w_j^{(0)}, \cdots w_n^{(0)}\right)$, $0 \leq w_j^{(0)} \leq 1$ and $\sum\limits_{j=1}^{n} w_j^{(0)} = 1$. The total weight of the attribute:

$$W^* = (w_1{}^*, \cdots, w_j{}^*, \cdots w_n{}^*) \tag{19}$$

$$w_j{}^* = \beta w_j^{(0)} + (1-\beta) w_j \sum_{j=1}^{n} w_j{}^* = \beta \sum_{j=1}^{n} w_j^{(0)} + (1-\beta) \sum_{j=1}^{n} w_j = 1$$

$\beta\,(0 \le \beta \le 1)$ is the weighted trade-off coefficient. If the ranking of the schemes is highly sensitive to weight changes, the reliability of the evaluation results is difficult to guarantee. It is also tricky for decision-makers to make choices [11]. To judge the rationality of the total weights, Judgement Theorem 2 is proposed.

**Judgement Theorem 2.** If the scheme ranking is less sensitive to changes in the total weight, then the total weight is relatively reasonable.

## 4 Scheme ordering steps

1. The distance from the first plan to the positive ideal plan is:

$$d_i{}^* = \sqrt{\sum_{j=1}^{n} (w_j{}^*)^2 (c_{ij} - c_j{}^*)^2} \tag{20}$$

2. The distance from the $i$ plan to the negative ideal plan is:

$$d_i{}^- = \sqrt{\sum_{j=1}^{n} (w_j{}^*)^2 (c_{ij} - c_j{}^-)^2} \tag{21}$$

3. The relative closeness of the $i$ scheme to the positive ideal scheme is:

$$D_i = \frac{d_i^-}{d_i^- + d_i^*} \tag{22}$$

   The larger the $i = 1 \cdots m$, $j = 1 \cdots n$, $D_i$ better than $i$ plan.

4. Arrange the pros and cons of the schemes in descending order of $D_i$ value.

5. Use the weighted trade-off coefficient $\beta$ to perform sensitivity analysis on the ranking of the schemes.

## 5 Case study

A company's production line needs to choose robots among the four submitted models. Now four suppliers are providing four solutions: $s_1$, $s_2$, $s_3$, $s_4$. Each program has six attributes [12]. The specific data are shown in Table 1. $u_5$ and $u_6$ are qualitative indicators. According to the relationship between fuzzy numbers and language variables, we use fuzzy triangular numbers and trapezoidal fuzzy numbers to represent:

**Table 1** The six attribute values of the four robots

| | $u_1$ | $u_2$ | $u_3$ | $u_4$ | $u_5$ | $u_6$ |
|---|---|---|---|---|---|---|
| $s_1$ | 2 | 2.5 | [55,56] | [94,114] | Normal (0.4, 0.5, 0.6) | Very high (0.85, 0.9, 0.95, 1) |
| $s_2$ | 2.5 | 2.7 | [30,40] | [84,104] | Low (0.2, 0.3, 0.4) | Normal (0.3, 0.4, 0.6, 0.7) |
| $s_3$ | 1.8 | 2.4 | [50,60] | [100,120] | High (0.6, 0.7, 0.8) | High (0.5, 0.6, 0.8, 0.9) |
| $s_4$ | 2.2 | 2.6 | [35,45] | [90,110] | Normal (0.4, 0.5, 0.6) | Normal (0.3, 0.4, 0.6, 0.7) |

Subjective weight $W^{(0)}$ = (0.2, 0.2, 0.1, 0.1, 0.2, 0.2). The weight compromise factor $\beta$ is 0.4. We use formulas (1)–(3) to standardise the evaluation matrix $A$ to obtain a standardised matrix $C$,

$$C^T = \begin{bmatrix} 0.4671\ 0.5839\ 0.4204\ 0.5139 \\ 0.4897\ 0.5289\ 0.4701\ 0.5093 \\ 0.5873\ 0.3704\ 0.5820\ 0.4233 \\ 0.5090\ 0.4600\ 0.5383\ 0.4894 \\ 0.4845\ 0.3101\ 0.6590\ 0.4845 \\ 0.6592\ 0.3833\ 0.5212\ 0.3833 \end{bmatrix} \tag{23}$$

The positive ideal solution is $A^*$ = (0.4204, 0.5289, 0.5873, 0.4600, 0.6590, 0.6592).

1. Use model (14). If we set the system parameter $\rho = 1$, we can get the objective weight W = (0.149, 0.1131, 0.1559, 0.1203, 0.2291, 0.2326). Then the comprehensive weight $W^* = \beta \times W^{(0)} + (1-\beta) \times W$ $W^*$ = (0.1694, 0.1479, 0.1335, 0.1122, 0.2175, 0.2196).
2. The distance from each plan to the positive ideal plan is $\begin{matrix} d_1^* = 0.0396\ d_2^* = 0.1050 \\ d_3^* = 0.0327\ d_4^* = 0.0765 \end{matrix}$, respectively.
3. The distance from each plan to the negative ideal plan is $\begin{matrix} d_1^- = 0.0797\ d_2^- = 0.0124 \\ d_3^- = 0.0908\ d_4^- = 0.0411 \end{matrix}$, respectively.
4. The relative closeness of each scheme to the positive ideal scheme is $\begin{matrix} D_1 = 0.6683\ D_2 = 0.1053 \\ D_3 = 0.7350\ D_4 = 0.3496 \end{matrix}$.
5. So, the sorting result: $s_3 \succ s_1 \succ s_4 \succ s_2$.
6. Sensitivity analysis. Sensitivity analysis observes the influence of the trade-off coefficient $\beta$ on the ranking of plans [13]. The results are shown in Table 2 and Figure 1.

When the system parameter is $\rho = 0.6$, the sensitivity analysis is shown in Figure 2.

2. Using model (8), we can get the objective weight W=(0.0777,0.5411,0.0392,0.3091,0.0157,0.0171). The sensitivity analysis is shown in Figure 3.

From Figure 3, the weight is more sensitive to the change of the program ranking, and it is difficult for decision-makers to choose. The main reason is that the objective weight is not very reasonable, and the evaluation value of each scheme under the second attribute is the most consistent. This causes the weight of the second attribute to be too large and it exceeds 50%. It is 35 times the minimum weight. In Figures 1 and 2, the pros and cons of the scheme are more pronounced, so it is easier for decision-makers to judge.

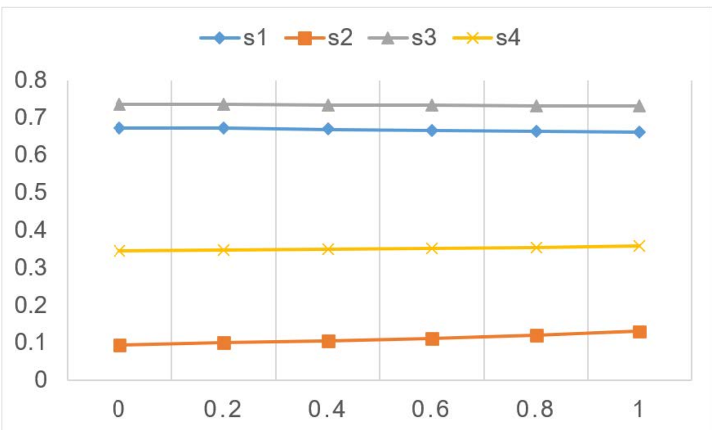


**Fig. 1** The influence of $\beta$ on the order of the schemes

**Table 2** $\beta$ influence on the closeness of each plan

| $\beta$ | ***s*1** | ***s*2** | ***s*3** | ***s*4** |
|---|---|---|---|---|
| 0 | 0.6726 | 0.0944 | 0.7366 | 0.3451 |
| 0.2 | 0.6705 | 0.0994 | 0.7358 | 0.3472 |
| 0.4 | 0.6683 | 0.1053 | 0.735 | 0.3496 |
| 0.6 | 0.666 | 0.1118 | 0.734 | 0.3522 |
| 0.8 | 0.6637 | 0.1191 | 0.7329 | 0.355 |
| 1 | 0.6613 | 0.1296 | 0.7316 | 0.358 |

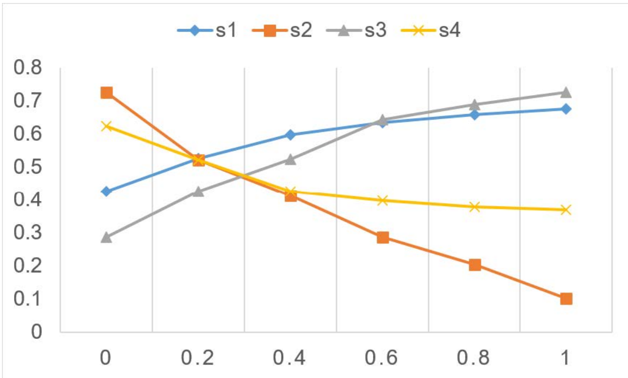


**Fig. 2** The influence $\beta$ on the ordering of the schemes

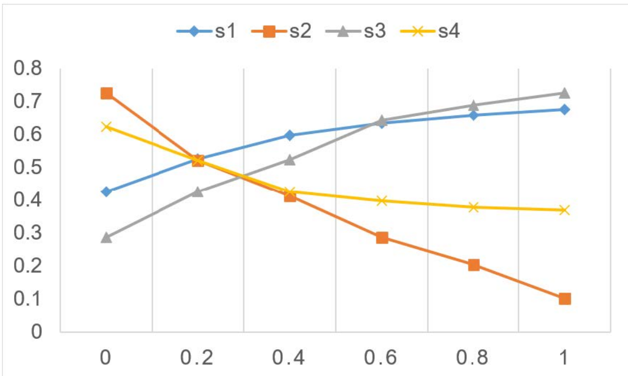


**Fig. 3** The influence $\beta$ on the order of the schemes

## 6 Conclusion

This article studied the mixed multi-attribute decision-making problem with quantitative and qualitative indicators and converts interval and fuzzy numbers into exact numbers to obtain a standardised judgement matrix. This method can resolve some of the issues involved with the mixed decision-making problem and simplify the calculation with the undefined index being the non-linear fuzzy number. We have established an entropy coefficient model for solving the objective weights of attributes. This model has a certain degree of flexibility, and the obtained weights are relatively reasonable.

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