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Abstract
The structure of Yablo's paradox is analysed and generalised in order to show that beginningless step-by-step determination processes can be used to provoke antinomies, more concretely, to make our logical and our ontological intuitions clash. The flow of time and the flow of causality are usually conceived of as intimately intertwined, so that temporal causation is the very paradigm of a step-by-step determination process. As a consequence, the paradoxical nature of beginningless step-by-step determination processes concerns time and causality as usually conceived.

Keywords
Antinomy; circularity; ungroundedness; determination structure, recursion.

I. Yablo 1993 presents an infinite sequence of sentences $s_1, s_2, s_3, \ldots s_n$, … each of them saying that all the sentences posterior in the sequence are untrue:

s_n: for all natural numbers $m>n$, s_m is untrue

This structure is paradoxical because there is no way to consistently assign a truth-value to any sentence in the sequence. So far, the situation is the same as in the Liar and Liar-like sentences: we are also incapable of consistently assigning truth-values to Liar-like sentences. In the case of Liar-like sentences the most widely accepted diagnose is that the kind of self-reference present in such sentences induces circularity in the process of truth-value determination. But none of Yablo's sentences seems to be self-referential, not even indirectly, or involve circularity.

In spite of this, Priest 1997 tries to find circularity in Yablo's sequence. On the other hand, Goldstein 2006 blames *underspecification* due to *ungroundedness* but not to circularity. I shall argue that Priest is

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wrong while Goldstein's suggestion points in the correct direction. Following Shackel 2005 I shall generalise Yablo's structure in order to present a class of situations in which ungroundedness elicits *conflicting intuitions.* Finally, I reject a solution proposed by Yablo 2000.

II. Priest 1997 made a serious attempt to find circularity in Yablo's liars. He finds circularity in the function s specifying the s_n:

> It is now the function s that is a fixed point: s is the function which, applied to any number, gives the claim that all claims obtained by applying s itself to subsequent numbers are not true. (Priest 1997: 239).

But the fact that Yablo's sequence is algorithmically enumerable shows that there is no vicious circularity in its specification. Priest thinks that circularity also shows up in the argument to paradox. Priest writes that the argument requires a step like:

for any *n:*

$Ts_n \Rightarrow \forall k > n, \neg Ts_k$ (*) (Priest 1997: 237)

where '*T*' denotes the truth predicate.

Now, the justification of (*) seems to be a T-schema like

(T*) $Ts_n \leftrightarrow \forall k > n, \neg Ts_k$

Priest claims that (*) is a consequence of no instance of a T-schema because the n in it is a free variable and a T-schema would only apply to sentences, not to open formulas. So Priest concludes that for the argument to go through we must generalise the T-schema to formulas with free variables and that this implies substituting a satisfaction predicate $S(n, s\cdot)$ for Ts_n in (T*), where $s\cdot$ stands for '$\forall k > x, \neg Ts_k$'. Then he claims that $s\cdot$ is circular since:

> $s\cdot$ is the predicate 'no natural number greater than x satisfies this predicate'. (Priest 1997: 238).

Bueno and Colyvan 2003 have shown that there is no need to apply a T-schema to open formulas to derive a paradox from Yablo's sequence. They derive a paradox by picking a particular Yablo sentence (s_1, in fact) and reasoning for it instead of arguing about some unspecified one, so they avoid the use of free variables. And indeed one can easily convince oneself that the reasoning about s_1 can be repeated for any sentence in Yablo's sequence.

Goldstein compares Yablo's sequence with an *ungrounded* Fibonacci series:

> .
> .
> .
> The sum of the two preceding numbers
> The sum of the two preceding numbers
> The sum of the two preceding numbers
> .
> .
> .
> (Goldstein 2006: 872)

and suggests that the kind of underspecification here evident also affects Yablo's sequence. I will try to show that in fact ungroundedness, under the form of a beginningless time or time-like process, leads in this context to incompatible intuitions.

III. Before shifting to a more general frame, I would like to outline the idea by means of a temporal version of Yablo's setup. Imagine a time without a beginning inhabited only by an infinite row of temporally successive thinkers with no first, each of which is in the absolute past of all the following thinkers and thinks or states only this: 'nobody has ever been right'. I will call these thinkers 'Yabloesque thinkers'.

The kind of circularity characteristic of Liar-like sentences is absent here. No Yabloesque thinker evaluates either directly or mediately his own statement, he evaluates only *past events.* While one can assume that, due to circularity, a sentence depending on itself to receive full determination may turn out to be underdeterminate, no sentence referring to past events may lack full determination because it grounds in fully determinate events, in events that are objectively settled by the objectivity that being in the past confers.

The past, as usually conceived, is *determinate* in the sense that Excluded Middle unproblematically applies to it. For any Yabloesque thinker *T* either no previous thinker was right or at least one of them was (*in particular, if none succeeded in making a statement, none was*) and whatever the facts are, they are objectively settled for *T* because they lie in the absolute past of *T; so the possible failure of any previous Yabloesque thinker to make a statement should not imply a corresponding failure of T,* where *T* is, of course, an arbitrary Yabloesque thinker.

Similarly, we can doubt whether Epimenides succeeded in making a statement by means of his famous utterance but if John states at present time that Epimenides did fail, John cannot fail himself, because either Epimenides failed or he did not, and the truth of this disjunction renders John's statement either objectively true or objectively false. So no sentence about past events, provided it is not too vague a sentence, can fail to have a truth-value. And there is no obvious vagueness in the assertions of our Yabloesque thinkers.

So it appears that, if the chain of the Yabloesque thinkers existed, each thinker in it would succeed in asserting a definite state of affairs to which Excluded Middle would apply; therefore, each thinker would succeed in making a statement with a definite truth-value. Since this cannot be so, the existence of the chain seems impossible. But the impossibility of the chain is the impossibility of a *beginningless* chain of Yabloesque thinkers: no *grounded* sequence of Yabloesque thinkers is paradoxical. Consider a first thinker's statement 'nobody has ever been right'; this seems literally true; hence all posterior thinkers must be wrong. There is no paradox at all.

But a beginningless chain of Yabloesque thinkers seems possible if only a beginningless time is possible. This granted, if we assume the possibility of a beginningless time, we can use the Yabloesque thinkers to arrive at *conflicting intuitions:* on the one hand, the determinateness of the past implies that each Yabloesque thinker must succeed in making a statement; on the other hand, logic reveals that none can do so. From the point of view of pure logic a beginningless time could be occupied by a chain of Yabloesque thinkers, provided each of them fails to make a statement. But, on the other hand, the determinateness of the past renders it absolutely incomprehensible how anyone could fail in making a statement when evaluating past events.

Of course, if we take both intuitions to be sound, we get a *reductio* of the possibility of a beginningless chain of Yabloesque thinkers, and from here it is a small step to the impossibility of a beginningless time.

Shackel 2006 has shown that Yablo's paradox instantiates a structure also to be found in Benardete 1964. The generalisation required to characterise this structure implies shifting from the concept of *temporal order* to the more general of *determination order;* the corresponding generalisation of temporal terms will be indicated by quotation marks: 'time', 'past', 'successive'. The purpose is to capture two crucial ontological features of time as usually conceived: the determinateness of the past and the determination of each instant by its past.

Define a *determination structure* as a quadruple ⟨S, V, R, f⟩ where:

S is the set of items whose values are to get determined.
V is a set of values for the members of *S*.
R is a *determination order* on *S*, which means that

1. *R* is a strict linear order on *S*.
2. *R* is *time-like*, that is, given an item *x*, the values of all items *R*-prior to *x* are determinate to the effect of the determination of the value of *x*, just as the past of an instant *t* is determinate for the determination of *t* by temporal causation.

f is a *determination function*, i.e. a function *f*: *S*→*V* whose domain can be ordered into a time-like chain ⟨*S*, *R*⟩, so that *f* can be thought of as giving values *'successively' along* ⟨*S*, *R*⟩ *by recursion*, i.e. for each item *x*, the value of *f(x)* depends *only* upon the values of *f* for some items *R*-prior to *x*, if *x* is not the *R*-first item, while *f* assigns *x* some fixed value if *x* is the *R*-first item. *f* performs a *determination process*.

Temporal causation is just an instance of the more general concept of determination process, an instance where *f* is of temporal-causal nature. Besides strict sense temporality, also atemporal causal relations — perhaps the relation obtaining between the fact that snow is white and the fact that 'snow is white' is true — and computation devices using recursion give rise to determination structures.

Note that *f* complies with an *irreversibility condition*. The irreversibility condition renders the determination process causally irreversible; as a consequence, for each item *x*, it renders all determination episodes *R*-prior to *x* irreversibly determinate. This gives us concrete ontological intuitions about *f*: *any f obeying the irreversibility condition* assigns a unique value to each non *R*-first item *x* on the basis of values that are irreversibly determinate at *x*, and a unique value to the *R*-first item, if it exists. Intuitively speaking, the time-likeness of *R* guarantees that the system contains no *determination gap* and the irreversibility condition, by prohibiting determination loops, ensures consistency, i.e. that there is no *determination glut*.

Let us now give *f* a Yabloesque shape. Let $V = \{0, 1\}$ and let *f* assign *1* to *x,* for all *x,* if and only if all items that are *R*-prior to *x* (if some exist) are assigned *0:*

$$(f)\ \forall x_{\in S}\ (f(x) = 1 \leftrightarrow \forall y_{\in S}\ (yRx \rightarrow f(y) = 0))$$

If *x* is the *R*-first item, the second member of *(f)* is vacuously true. Then *f(x)=1* and the value of all subsequent items is *0.* But if there is no *R*-first item, we get the following conflicting intuitions (A) and (B):

(A) *f* consistently determines the value of all items up to an item *x,* for all *x.*

Indeed, *f* consistently determines the value of an item *x* if the values of all items *R*-prior to *x* are irreversibly determinate, and these are in fact so for each *x,* for they lie in the irreversibly determinate 'past' of *x*.

(B) *f* consistently determines the values of all items up to an item *x*, for **no** *x*.

Note that (A) is an ontological intuition decisively involving our conception of time and time-like processes while (B) is simply a logical one. (A) is the intuition that

(a) no step-by-step determination process can fail, so that
(b) the existence of determination processes without a first item implies the existence of *successful though ungrounded* step-by-step determination processes.

Consequently, the assumption of a determination process with no first item arouses incompatible intuitions concerning the Yabloesque function *f.*

IV. The case involves no purely logical contradiction. From a merely logical point of view all we have is that time-like determination processes cannot both have a beginningless 'past' and be ruled by a Yabloesque *f.* This is essentially what Shackel calls 'the unsatisfiable pair diagnosis' (Shackel 2005: 401). But this does not solve the antinomy, for there is in addition the ontological evidence that all items should be determined by *f* even from a beginningless 'past', if only it exists. If we dismissed this evidence on the grounds of the conflicting logical intuition, we would be concealing a genuine antinomy by conflating the logical and the ontological perspectives.

Using the logical evidence to fix the ontological intuition is what Yablo 2000 seems to attempt in a structurally identical context. I will try to show that this cannot work. Drawing on a formulation of Benardete's paradox by Priest 1999, Yablo presents an infinite non well-founded chain of demons, each of which intends to say 'yes' if all previous demons have said 'no', and 'no' otherwise. Yablo writes:

> If we focus on any particular demon, there is nothing to stop her from executing her intention, given the opportunity. All she has to do is call YES if her predecessors have called NO, otherwise NO. Does it follow that there is nothing to stop the demons from fulfilling their intentions as a group? Logic stops them. (Yablo 2000: 150).

The problem is that logic is no causal force that could intervene as an overall ontological factor to stop the demons. To see how unlike any ontological factor logic is, just ask *exactly which demons are stopped by logic,* for there is no logical necessity that a particular group of them be. Perhaps Yablo means that logic prohibits such a chain of successful demons from existing, not that it could stop them, if they existed. But this would in turn miss the ontological intuition that the chain of successful demons can in fact exist if only an infinite chain of demons with no first can. Yablo concludes:

> If there's a paradox here, it lies in the difficulty of combining individually operational subsystems into an operational system. But is this any more puzzling than the fact that although I can pick a number larger than whatever number you pick, and vice versa, we can't be combined into a system producing two numbers each larger than the other? (Yablo 2000: 151).

Again this overlooks the ontological intuition that, if each demon can execute her intention, given the opportunity, all can: if a time-like determination function can successively determine each item in a chain, it can eventually determine all the items in the chain; and if beginningless 'time' is conceivable at all, the function can be conceived of as performing its task from a beginningless 'past'.

So, given the possibility of a beginningless 'past', our ontological intuition sees no possibility for the demons (or the Yabloesque thinkers) to fail while our logical intuition sees that some must fail. We must either reject our ontological intuition as unreliable or reject the possibility of a beginningless determination process.

Logic should stand.[1]

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