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Improvement of Pilot Symbol Orthogonal Sequences in 2×2 to 4×4 MIMO Wireless Communication Systems with Channel State Estimation

Vadim Romanuke* (*Professor, Polish Naval Academy, Gdynia, Poland*)

***Abstract* – MIMO wireless communication systems with channel state estimation, in which 2 to 4 transmit-receive antenna pairs are employed, are simulated. The channel estimation is fulfilled by the orthogonal pilot signal approach, where the Walsh Hadamard-ordered sequences are commonly used for piloting. The signal is modulated by applying the quaternary phase shift keying method. Maximum 250 000 packets are transmitted through flat-fading Rayleigh channels, to which white Gaussian noise is added. Based on simulating 10 subcases of the frame length and number of pilot symbols per frame, it is ascertained that pilot symbol orthogonal sequences in 2×2 to 4×4 MIMO systems can be improved by substituting Walsh functions with partially unsymmetrical binary functions constituting the eight known orthogonal bases. The benefit is that the bit-error rate is substantially decreased, especially for 2×2 MIMO systems. Considering three cases of the pilot signal de-orthogonalization caused by two indefinite and definite pilot sequence symbol errors, the relative decrement varies from 0.123 % to 14.7 %. However, the decrement becomes less significant as the number of transmit-receive antenna pairs is increased.**

***Keywords* – Bit-Error rate; Channel estimation; MIMO; Orthogonal pilot sequences; Wireless communication.**

I. Introduction to MIMO with Channel Estimation

In modern wireless communication systems, for sending and receiving more than one data signal simultaneously over the same radio channel, a technique of the multiple input and multiple output (MIMO) is used. This is implemented for multiplying the capacity of a radio link, where the multipath propagation is exploited. Nevertheless, in fact, MIMO is a technique using multiple antennas at the transmitter and receiver ends of a wireless communication system [1]. This is why MIMO systems are used in wireless communication standards, including IEEE 802.11n (Wi-Fi), IEEE 802.11ac (Wi-Fi) [2], HSPA+ (3G), WiMAX, Long Term Evolution (4G LTE) [3], [4], and 5G by implementing massive MIMO [5], [6]. MIMO technology has been officially standardized for wireless LANs, 3G and 4G mobile phone networks, but it is being increasingly tested and spread in many other wireless communication standards.

To sustain high quality of links, MIMO operates on channel state information (CSI) [1], [3], [5], [6]. The CSI is required to know how a signal propagates from the transmitter to the receiver for adapting transmissions to current channel conditions, which is crucial for achieving reliable communication with high data rates in multiantenna systems. As the CSI represents the combined effect of scattering, fading, and power decay with distance, it is not known at the receiver in a realistic scenario, so the CSI is extracted from the received signal. The channel estimator can perform this task by using orthogonal pilot signals prepended to every packet [5]. Compared to a blind approach, where the channel estimation is based only on the received data, without any known transmitted sequence, the tradeoff is the accuracy versus the overhead. The orthogonal pilot signal approach has a higher overhead than the blind approach, but it achieves a better channel estimation accuracy than the blind approach [1], [5].

In practice, the coherence time of the channel limits the length of orthogonal pilot sequences, from which the channel between the transmitter and receiver is estimated [7]. Besides, the reuse of pilot sequences of several co-channel cells leads to pilot contamination that worsens the MIMO performance [1], [3], [5], [6].

Another problem is that a loss of a symbol (this is, in other words, a symbol error) in a pilot sequence (due to channel noise and interference) leads to the pilot signal de-orthogonalization (PSdeO). Obviously, the PSdeO also worsens the MIMO performance [4], [5]. While being in an area of weaker signals, the cumulative effect of these negative effects is practically perceived as frequent fading distortions (e.g., bouncing antenna signal indicator) [3], [4], [7].

II. Motivation

In MIMO, as well as in other wireless communication systems exploiting orthogonality, orthogonal codes are generated based on Walsh functions [1]. In particular, Walsh functions are generated from the Hadamard matrix [7]. Thus, the first orthogonal sequence of pilot symbols is usually the sequence of ones, which is the Walsh function of the zeroth order (being a function-constant) [8]. The second orthogonal

* Corresponding author
E-mail: romanukevadimv@gmail.com

sequence of pilot symbols is the Walsh function of the last order (in the given finite binary basis ordered by Walsh [7], [9]), which is an ideal meander of the highest frequency. In general, the structure of Walsh functions is symmetrical (considering from the middle of the unit interval on which the functions are defined) [7], [8]. Partially unsymmetrical binary functions (PUBFs) which constitute orthogonal bases are also known (e.g., see [7]). The eight orthogonal bases of such binary functions [9] were simulated to substitute the respective Walsh functions in wireless communication systems with the code division multiple access (CDMA). It was shown in [10] that such orthogonal sets of binary functions could outperform a Walsh set. Namely, using PUBFs instead of Walsh functions allows decreasing the bit-error rate (BER) by 3 % to 5 % [10]. It is thereafter assumed that BER in MIMO systems with the orthogonal pilot signal approach can be decreased by similarly substituting Walsh functions with PUBFs. Obviously, this is believed to increase throughput and reliability of MIMO links by mitigating effects of PSdeO and pilot contamination [5].

III. Goal and Steps to Achieve It

Issuing from the plausibility to improve orthogonal sequences of pilot symbols, the goal is to estimate the BER performance of 2×2, 3×3, and 4×4 MIMO systems with channel state estimation by the orthogonal pilot signal approach for both the Walsh (Hadamard) and PUBFs. Along with the case of perfect orthogonality, the BER performance should be also estimated for various scenarios of PSdeO. To achieve the goal, 2×2 to 4×4 MIMO wireless communication systems with channel state estimation are to be simulated, where the number of transmit and receive antennas is the same (two, three, and four, respectively). The simulation will be configured and carried out by using MATLAB® R2019a Communications System Toolbox™ (CST) functions. The BER performance is to be plotted versus the bit-energy-to-noise-density ratio (BENDR or, as it is often referred to, "Eb/No") in a wide range starting from 0 dB with a step of 1 dB. A subsequent interpretation of the simulation results must reveal whether the improvement of pilot symbol orthogonal sequences is real and what the benefit is.

IV. Configuration of the Simulation

For using 2 to 4 transmit antennas, the signal is modulated by applying the quaternary phase shift keying (QPSK) method. In the CST, this is realised by the *QPSKModulator* object. The modulated signal is then encoded by using the CST *OSTBCEncoder* object. The *OSTBCEncoder* object encodes an input symbol sequence using orthogonal space-time block code (OSTBC). The block maps the input symbols block-wise and concatenates the output codeword matrices in the time domain. It is worth noting that the symbol rate of the code is 1 for a 2×2 system, and is 3/4 for 3×3 and 4×4 systems. Thus, the frame length denoted by F is set at 36, 72, 144, 288 symbols. Subsequently, the number of pilot symbols per frame denoted by P, which commonly does not exceed 25 % of the frame length, is set at integers shown in Table I, with respect to each frame length. This results in the 10 subcases of the two parameters paired for simulation.

It is reasonable to set the maximum BENDR at 8 dB. Thus, the simulation is to be run over the BENDR range from 0 dB to 8 dB with a step of 1 dB. For each of those 9 BENDR points, maximum 250 000 packets are transmitted through flat-fading Rayleigh channels [11], to which white Gaussian noise is added by applying the CST *AWGNChannel* object. It is assumed that the channel remains unchanged for the length of the packet (i.e., it undergoes slow fading), and the channel undergoes independent fading between the multiple transmit-receive antenna pairs [12].

TABLE I
The 10 Subcases of the Frame Length and Number of Pilot Symbols per Frame

Frame length (F)	288	144	72	36
Number of pilot symbols per frame (P)	64	32	16	8
	32	16	8	
	16	8		
	8			

The CST *OSTBCCombiner* object combines the signals from all of the receive antennas and the channel estimate signal to extract the soft information of the symbols encoded by an OSTBC. The combining algorithm uses only the estimate for the first symbol period per codeword block. The output of the combiner is demodulated by applying the CST *QPSKDemodulator* object.

While an end-to-end MIMO system is simulated, the number of errors is registered. The maximum number of errors is 10 % of the maximum number of packets. Thus, if 25 000 errors occur at a given BENDR, the simulation loop is broken (for the given value of BENDR).

V. Cases to Be Simulated

The above-mentioned PSdeO occurs when the negative value (which might be thought of as logical "0" or symbol "0") of a binary function is switched into the positive value (logical "1" or symbol "1") and vice versa. These pilot sequence symbol errors are notationally referred to as "0→1" and "1→0", respectively. To estimate the BER performance under circumstances of PSdeO, the six cases (Table II) are to be simulated for orthogonal codes built by both the Walsh and PUBFs.

In an N×N MIMO system, the N pilot sequences are taken as the first N Walsh Hadamard-ordered functions from the basis of P functions (Fig. 1). In the case of basing orthogonal codes on PUBFs, the N pilot sequences are taken as the last N PUBFs from each of the eight bases of P functions (Fig. 2).

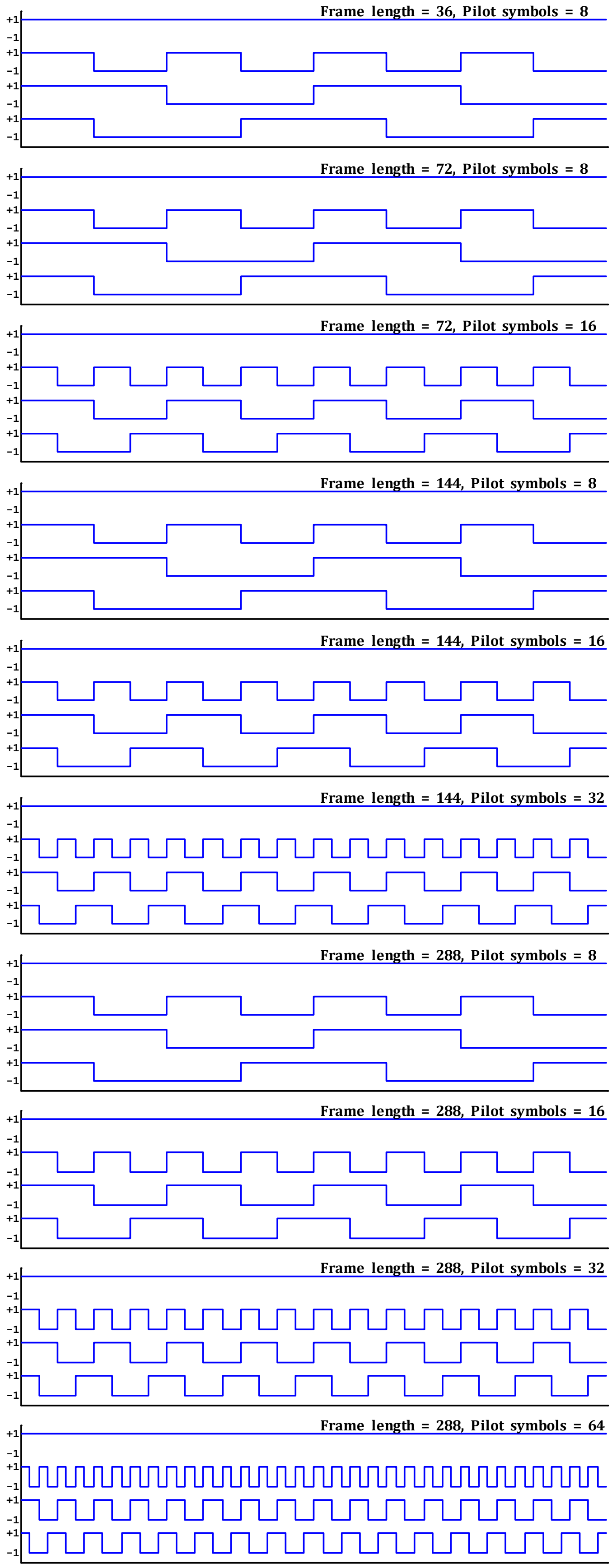

Fig. 1. The four Walsh Hadamard-ordered orthogonal functions (sequences) in a 4×4 MIMO system for the 10 subcases of the frame length and pilot symbols per frame (Table I). The frequency of the third and fourth meanders (differed only by a symbol shift) is half the frequency of the second meander.

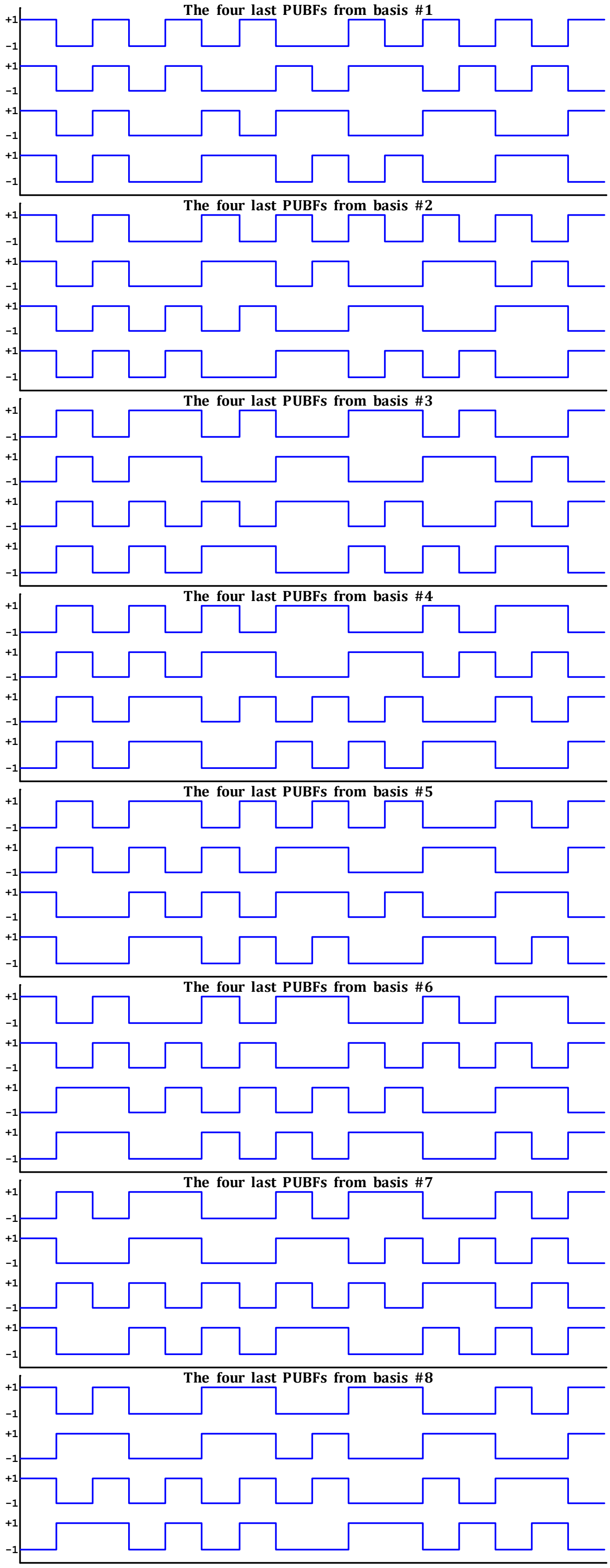

Fig. 2. The four last PUBFs (sequences) from each of the eight bases of 16 functions in a 4×4 MIMO system for the frame of 72, 144, or 288 symbols, where 16 pilot symbols per frame are used). Unlike the 4×4 MIMO system with Walsh sequences, the meanders here have roughly the same frequency.

TABLE II
THE SIX CASES OF SIMULATION FOR BOTH THE WALSH AND PUBFS

Simulation case #	Description of the case	Notation of pilot sequence symbol errors
1	Perfect orthogonality	there are no pilot sequence symbol errors
2	PSdeO caused by one indefinite pilot sequence symbol error	“0→1” or “1→0”
3	PSdeO caused by one definite pilot sequence symbol error	“0→1”
4	PSdeO caused by two indefinite pilot sequence symbol errors	“0→1” and “0→1”, or “1→0” and “1→0”, or “1→0” and “0→1”, or “0→1” and “1→0”
5	PSdeO caused by two repeated definite pilot sequence symbol errors	“0→1” and “0→1”
6	PSdeO caused by two repeated indefinite pilot sequence symbol errors	“0→1” and “0→1” or “1→0” and “1→0”

VI. SIMULATION RESULTS

In order to present simulation results easily readable and comparable, the BER performance is visualized for every simulation case in the same figure by using subplots for every subcase of the frame length and number of pilot symbols per frame (see Table I). Thus, the BER for 2×2, 3×3, and 4×4 MIMO systems is plotted on the same axes using different line thickness (and, additionally, marker size): thicker lines (along with larger markers) correspond to the system with a greater number of transmit-receive antenna pairs. In the case of basing orthogonal codes on Walsh functions, the BER performance markers are square dots (black colour); in the case of using PUBFs, the markers are circle dots (red colour).

Simulation results for the case of the perfectly orthogonal pilot sequences (simulation case #1) are presented in Fig. 3. It is clearly seen that the BER performance is better for the MIMO system with a greater number of transmit-receive antenna pairs. Thus, transmitting 288-symbol frames piloted with 8 symbols at BENDR of 0 dB allows achieving a BER of about 0.25 (i.e., every fourth bit is lost) for the 2×2 MIMO, whereas it is decreased to 0.19 and 0.15 for 3×3 and 4×4 MIMO systems, respectively. Herein, it is worth noting that for the 2×2 MIMO, at BENDR of up to 2 dB, the BER becomes slightly less by using PUBFs than that by using Walsh functions. Such a feature can be spotted for other two subcases of piloting with 8 symbols (when transmitting frames of 72 and 144 symbols). A similar low-BENDR BER decrement is partially seen at

$$F = 72 \text{ and } P = 16,$$
$$F = 144 \text{ and } P = 32,$$
$$F = 288 \text{ and } P = 16,$$
$$F = 288 \text{ and } P = 32,$$
$$F = 288 \text{ and } P = 64.$$

In the remaining two subcases, where

$$F = 32 \text{ and } P = 8,$$
$$F = 144 \text{ and } P = 16,$$

using PUBFs does not improve the BER performance.

In the case of one indefinite pilot symbol error (simulation case #2), PSdeO has its bad impact on the BER performance (see Fig. 4). For instance, in transmitting 288-symbol frames piloted with 8 symbols, the BER increases by approximately 0.02 at BENDR of 0 dB. Low-BENDR BER decrement, when PUBFs are used in 2×2 MIMO systems, similar to that in the case of the perfect orthogonality still can be spotted. However, the benefit is too tiny to be visually confirmed. In general, the BER polylines in Fig. 3 are almost replicated by those in Fig. 4 with the exception of that the BER performance significantly worsens (especially at low BENDRs) for 2×2 MIMO systems, and 3×3 and 4×4 MIMO systems with 8 pilot symbols. Besides, 3×3 and 4×4 MIMO systems, where

$$F = 72 \text{ and } P = 16,$$
$$F = 288 \text{ and } P = 16,$$

are badly influenced (at low BENDRs) by this PSdeO case also. On the contrary, 3×3 and 4×4 MIMO systems, where

$$F = 144 \text{ and } P = 16,$$
$$F = 144 \text{ and } P = 32,$$
$$F = 288 \text{ and } P = 32,$$
$$F = 288 \text{ and } P = 64,$$

are not influenced by that, and the respective BER polylines factually duplicate those in Fig. 3.

The case, in which PSdeO is caused only by switch “0→1” (simulation case #3), is unexpectedly revealed to be just as bad (see the respective BER performance in Fig. 5) as case #2. Once again, the BER performance for 2×2 MIMO systems significantly worsens at low BENDRs, whereas 3×3 and 4×4 MIMO systems with 16 pilot symbols or more are almost not affected by this PSdeO case at BENDRs greater than 2 dB.

In the case of two indefinite pilot sequence symbol errors (simulation case #4), the bad impact of PSdeO on the BER performance is more noticeable (Fig. 6). It is easily noticed that, for 2×2 MIMO systems with 8 pilot symbols, the BER becomes significantly smaller (by 0.005 to 0.01) by using PUBFs than that by using Walsh functions. The respective four subcases are seen in Fig. 6 without zoom-in. Moreover, piloting by PUBFs improves the BER even at high BENDRs. Nevertheless, using PUBFs almost does not differ from using Walsh functions in 3×3 and 4×4 MIMO systems affected by this PSdeO case.

As it is expected, the case, in which PSdeO is caused by two repeated switches “0→1” (simulation case #5), appears to be quite substantial (Fig. 7). Now, it is clearly seen that, whichever the number of transmit-receive antenna pairs is, the BER becomes significantly smaller by using PUBFs for the MIMO systems with 8 pilot symbols. The BER is decreased by up to 0.04 for the 2×2 MIMO systems, although the decrement is not greater than 0.01 for the 3×3 MIMO systems. Obviously, the BER decrement is much smaller for the 4×4 MIMO systems (it is hardly noticeable for the subcase of F = 288 and P = 8). Furthermore, the BER becomes significantly smaller (by 0.005 to 0.01) by using PUBFs for the 2×2 MIMO systems with 16 pilot symbols.

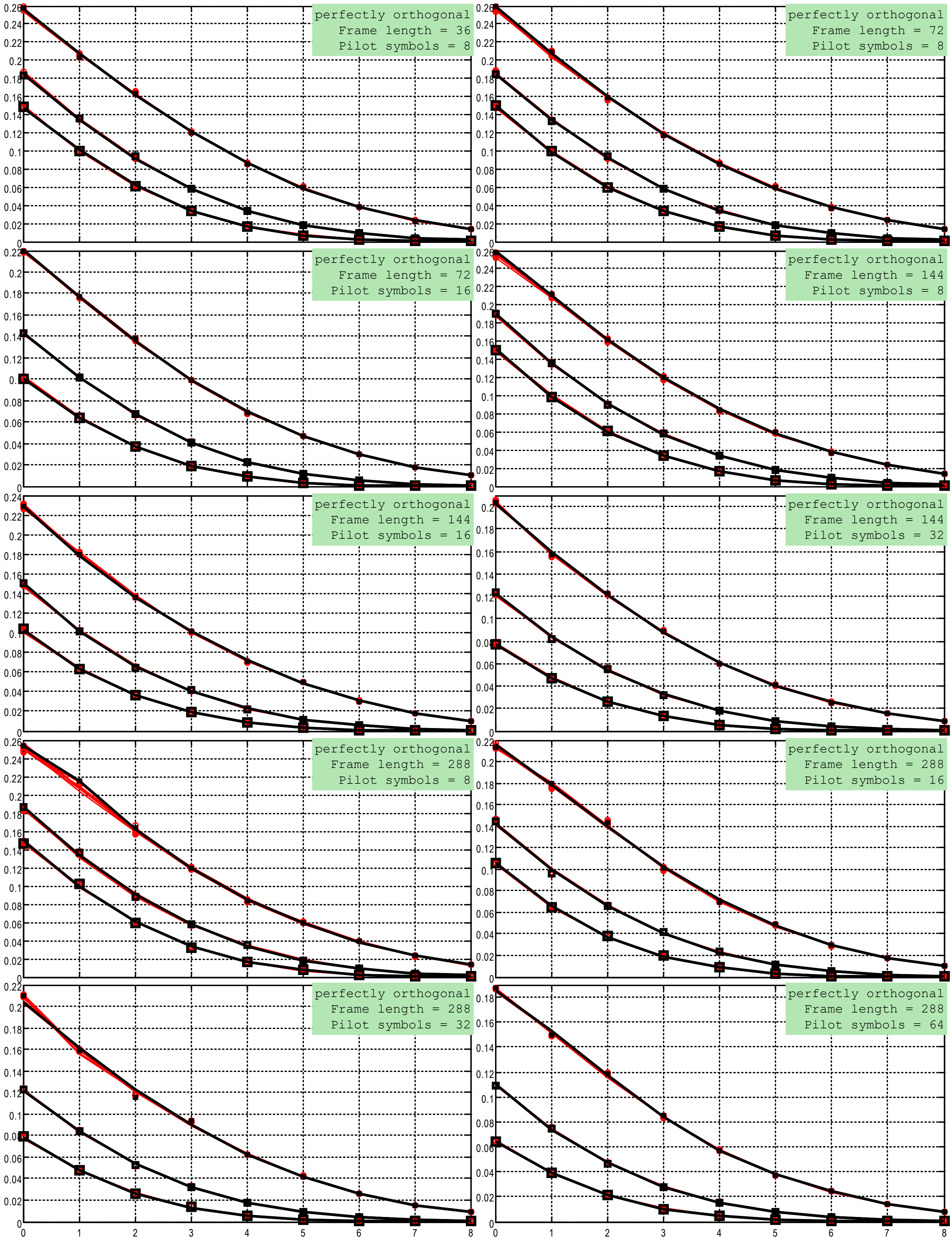

Fig. 3. The BER performance versus BENDR in the case of perfect orthogonality (simulation case #1). The BER performance is best for the 4×4 MIMO systems.

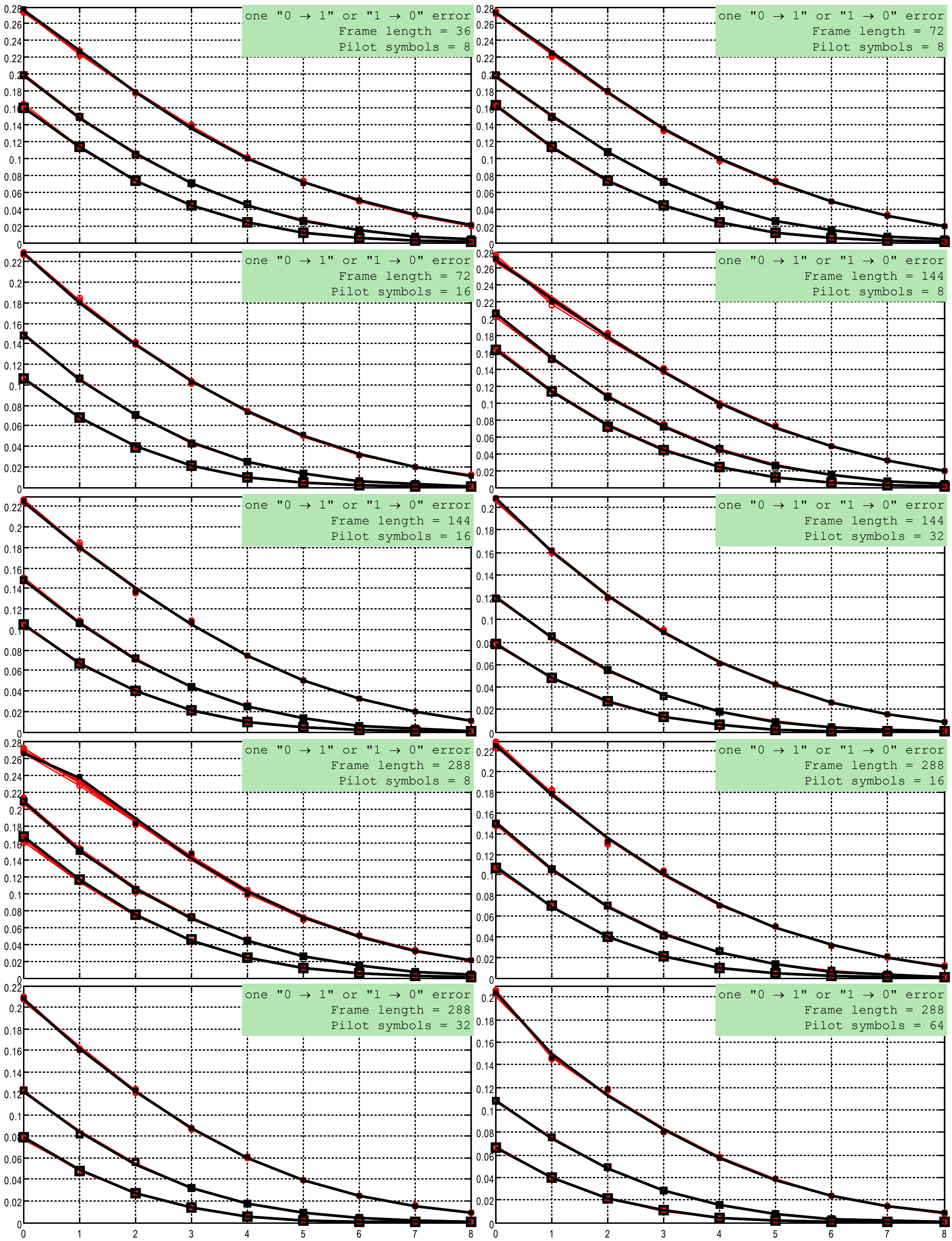

Fig. 4. The BER performance versus BENDR in the case of one indefinite pilot symbol error (“0→1” or “1→0”). This is PSdeO by simulation case #2.

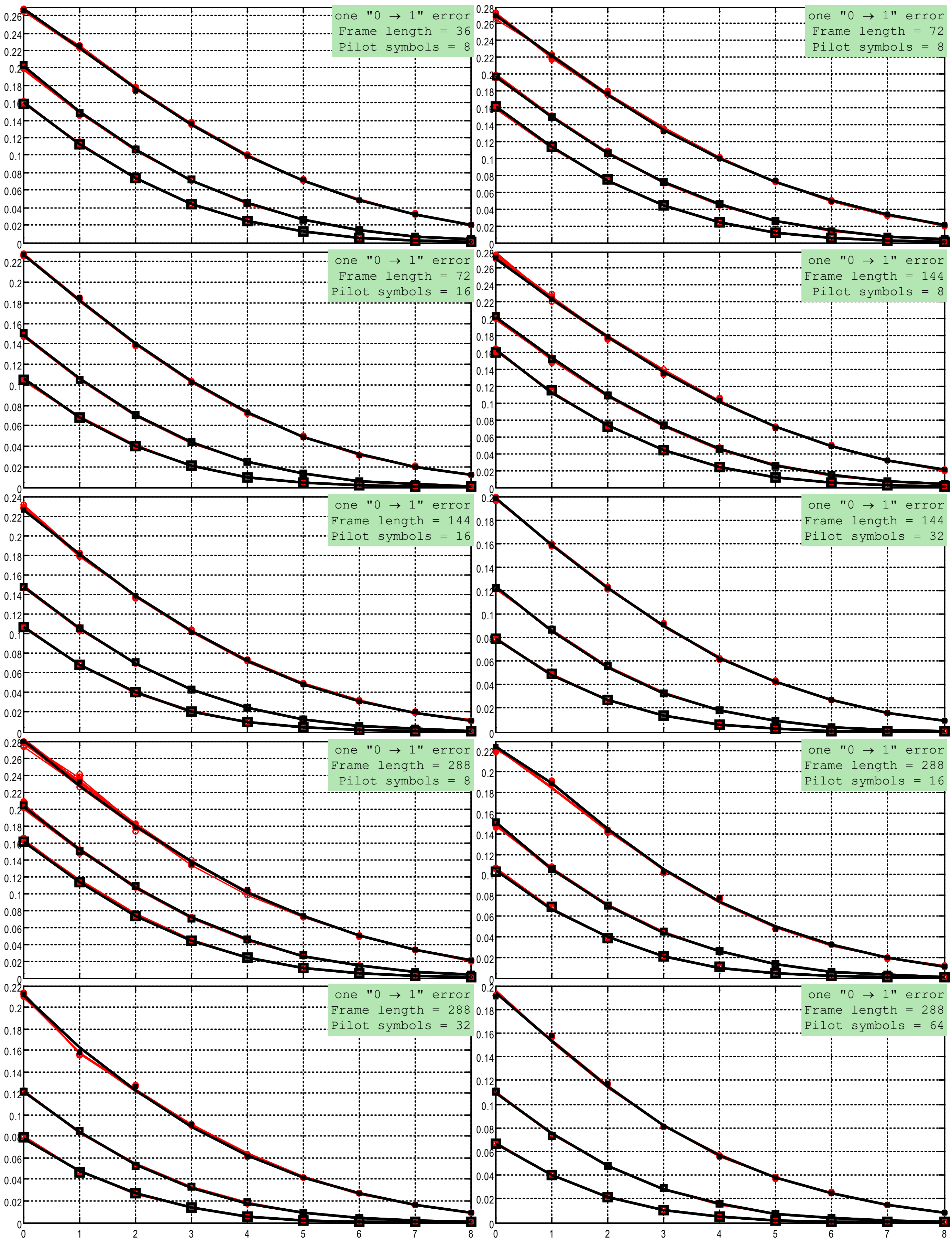

Fig. 5. The BER performance versus BENDR in the case of one definite pilot sequence symbol error ("0→1"). This is PSdeO by simulation case #3.

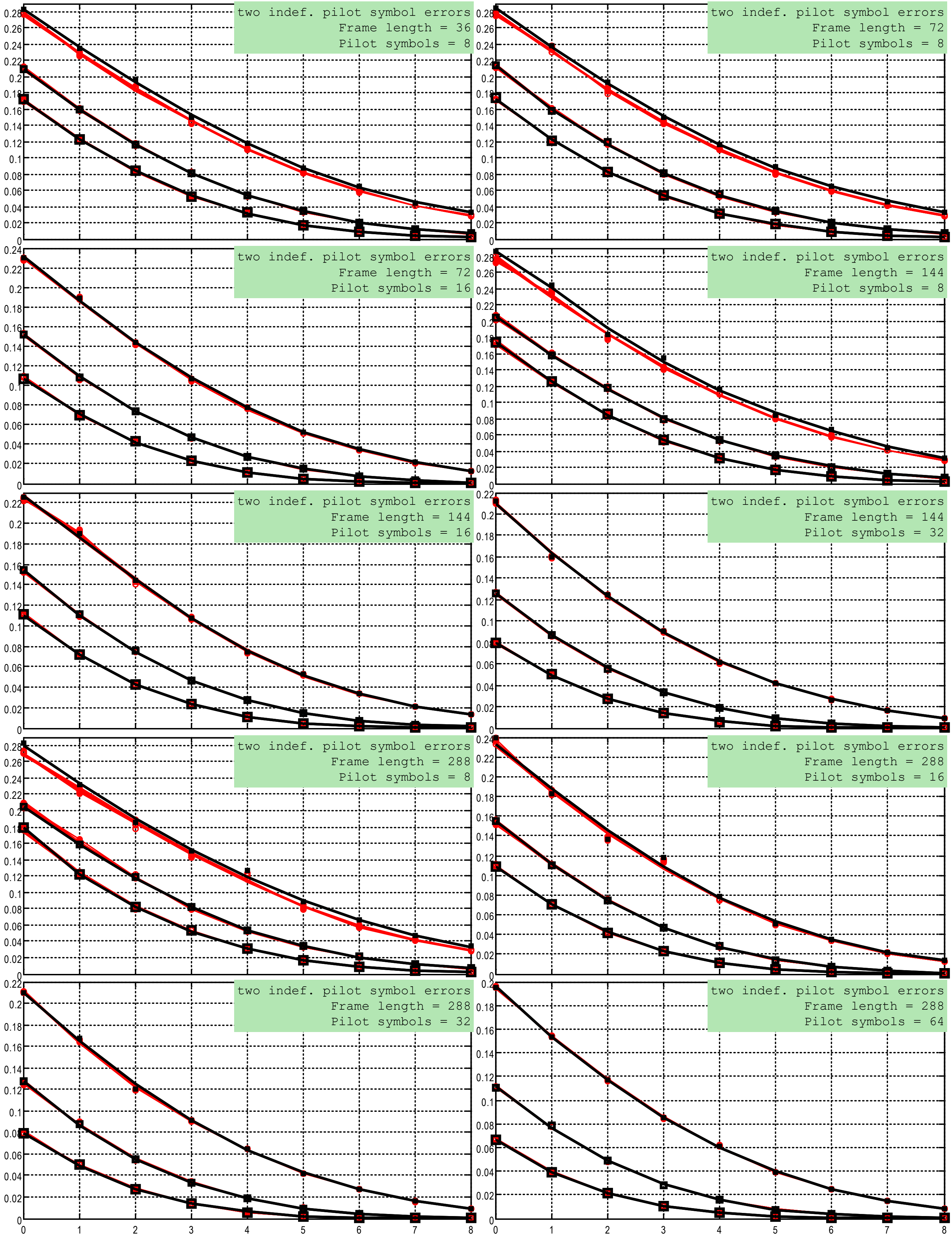

Fig. 6. The BER performance versus BENDR in the case of two indefinite pilot sequence symbol errors (see Table II). This is PSdeO by simulation case #4.

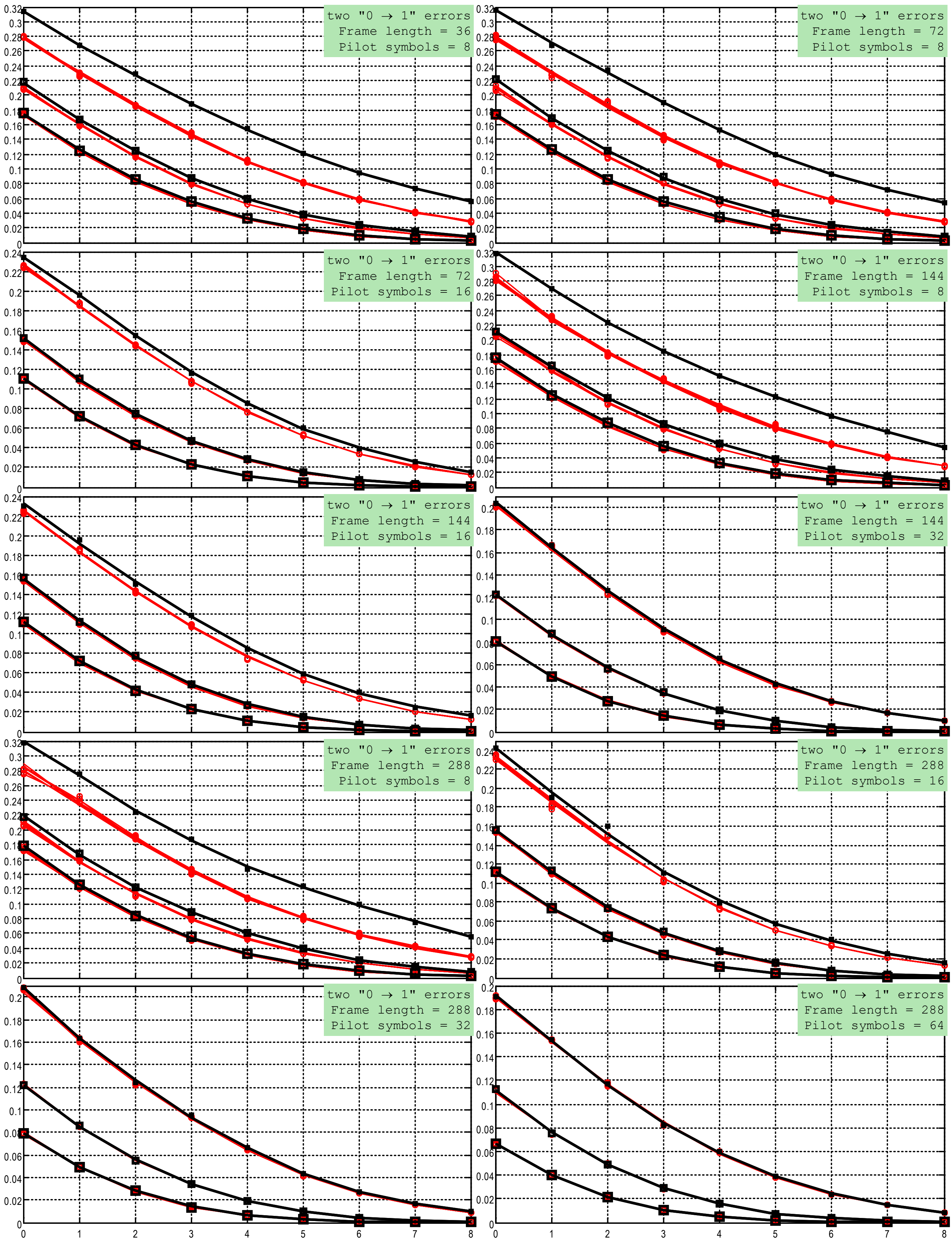

Fig. 7. The BER performance versus BENDR in the case of two repeated pilot sequence symbol errors (“0→1” and “0→1”). This is PSdeO by simulation case #5.

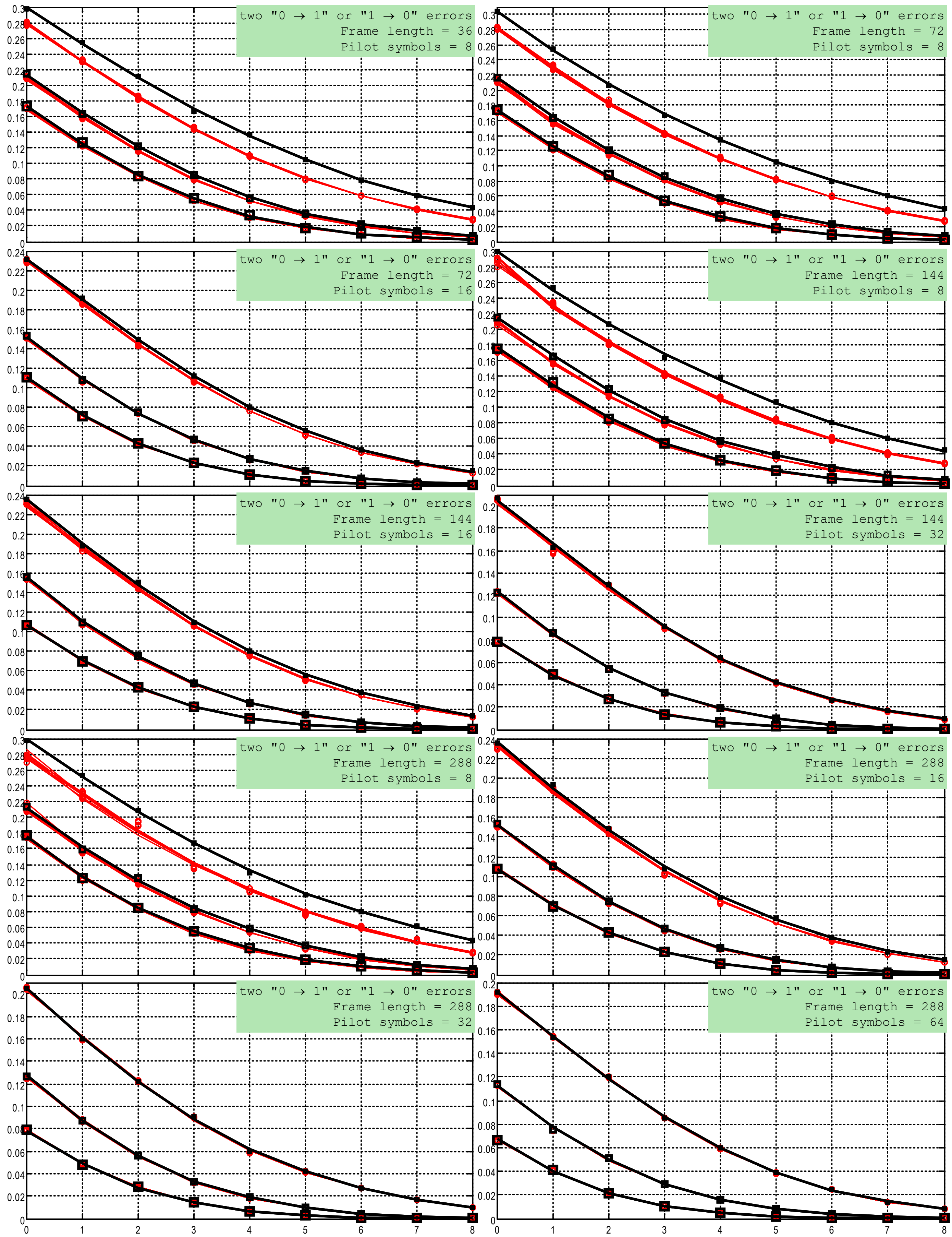

Fig. 8. The BER performance versus BENDR in the case of two repeated indefinite pilot sequence symbol errors (see Table II, PSdeO by simulation case #6).

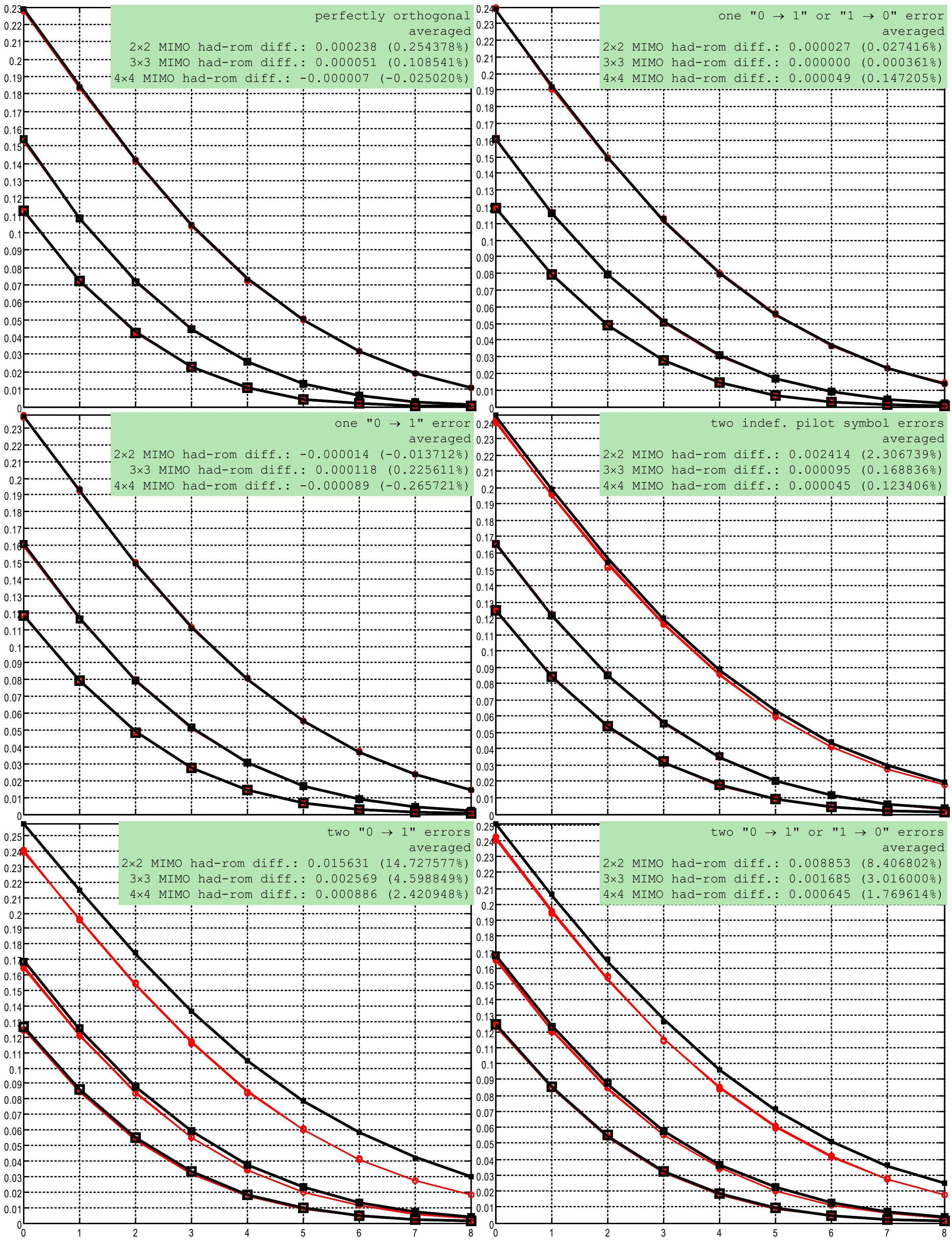

Fig. 9. The averaged BER performance versus BENDR for the six simulation cases. The BER performance difference is the greatest in the bottom subplot row.

The results of simulating PSdeO caused by two repeated indefinite pilot sequence symbol errors (simulation case #6) appear similar (Fig. 8) to the previous case results. The only difference is that now, when the repeated pilot symbol switch is indefinite, the BER decrement is less than that in the case of two repeated switches “0→1”.

The BER performance averaged over each of the six simulation cases is presented in Fig. 9 along with the mean percentage of the difference between the performance by Walsh Hadamard-ordered functions and PUBFs. These polylines show that the BER is undoubtedly decreased in the PSdeO cases caused by two pilot sequence symbol errors for 2×2 MIMO systems. Besides, in the PSdeO cases caused by two repeated pilot sequence symbol errors (see the subplot bottom row of Fig. 9) the BER decrement is clearly seen for 3×3 MIMO systems. Furthermore, in these subcases, the performance of 4×4 MIMO systems by using PUBFs is improved by 2.42 % (case #5) and 1.77 % (case #6).

VII. Discussion

As it has been mentioned above, the BER performance is improved as the number of MIMO transmit-receive antenna pairs is increased. In addition, the difference between the BER performance by Walsh Hadamard-ordered functions and PUBFs becomes smaller, which is confirmed by the bottom subplot row in Fig. 9, as well as by the respective subplots in Figs. 6–8, where the polylines are like being converged at a lower line (implying the least BER for 4×4 MIMO systems).

TABLE III
Recapitulation of the BER Performance by Walsh Hadamard-Ordered Functions and PUBFs

Case #	Description of the PSdeO case and its probability	Notation of pilot sequence symbol errors	Difference between the performance by Walsh Hadamard-ordered functions and PUBFs		
			2×2 MIMO	3×3 MIMO	4×4 MIMO
2	one indefinite pilot sequence symbol error, 0.5	“0→1” or “1→0”	0.027 % (unreliable)	0.00036 % (unreliable)	0.147 % (unreliable)
3	one definite pilot sequence symbol error, 0.25	“0→1”	–0.014 % (unreliable)	0.226 % (unreliable)	–0.266 % (unreliable)
4	two indefinite pilot sequence symbol errors, 0.5	“0→1” and “0→1”, or “1→0” and “1→0”, or “1→0” and “0→1”, or “0→1” and “1→0”	2.307 %	0.169 %	0.123 %
5	two repeated definite pilot sequence symbol errors, 0.125	“0→1” and “0→1”	14.728 %	4.5988 %	2.421 %
6	two repeated indefinite pilot sequence symbol errors, 0.25	“0→1” and “0→1” or “1→0” and “1→0”	8.407 %	3.016 %	1.7696 %

It is hard to claim what the probability of the perfect orthogonality case and PSdeO. However, PSdeO itself is likely and the five various scenarios of PSdeO studied above are very likely outcomes of transmitting signals through realistic environments. These scenarios are not equiprobable, though. Their theoretic probabilities (by a presumption of that cases #2 and #4 are the exhaustive events) are given in Table III along with the mean percentage of the difference between the BER performance by Walsh Hadamard-ordered functions and PUBFs (Fig. 9). The percentage for cases #2 and #3 are treated unreliable because the difference does not disappear as the number of MIMO transmit-receive antenna pairs is increased. The probability of case #5 is not small, so the performance of the 4×4 MIMO system can be significantly improved as the difference in 2.421 % is very significant, let alone 2×2 and 3×3 MIMO systems.

VIII. Conclusion

Based on the MATLAB simulations carried out, it is certain that pilot symbol orthogonal sequences in 2×2 to 4×4 MIMO systems with channel state estimation can be improved by substituting Walsh functions with PUBFs. The benefit is that the BER is substantially decreased, especially for 2×2 MIMO systems. Considering the cases of PSdeO with two pilot sequence symbol errors, which are 50 % probable, the relative BER decrement varies from 0.123 % to 14.7 %. In spite of the fact that the BER decrement becomes less significant as the number of transmit-receive antenna pairs is increased, the substitution is a promising way to increase throughput and reliability of MIMO links.

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Vadim V. Romanuke was born in 1979. He graduated from the Technological University of Podillya in 2001. The higher education was received in 2001. In 2006, he received the Degree of Candidate of Technical Sciences in Mathematical Modeling and Computational Methods. The degree of Doctor of Technical Sciences in Mathematical Modeling and Computational Methods was received in 2014. In 2016, Vadim Romanuke received the academic status of Full Professor.

He is a Professor of the Faculty of Mechanical and Electrical Engineering at the Polish Naval Academy. His current research interests concern wireless communication systems, job scheduling, semantic image segmentation, decision making, game theory, statistical approximation, and control engineering based on statistical correspondence.

Address for correspondence: 69 Śmidowicza Street, Gdynia, Poland, 81-127.
E-mail: romanukevadimv@gmail.com
ORCID iD: https://orcid.org/0000-0003-3543-3087