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Application of Benford's law to detect signs of under-invoicing in companies in the restaurant sector during the COVID-19 pandemic

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Abstract: The main objective of this study is to detect signs of under-invoicing by applying Benford's law to the Portuguese restaurant sector during the COVID-19 pandemic, in the context of government support policies. Between 2020 and 2021, the State adopted several measures to provide additional support to companies that have seen a significant decrease in their activity, namely, a reduction of at least 25% in turnover. A literature review was carried out focusing on the impact of the COVID-19 pandemic on the companies under analysis, the support measures adopted by the State and, finally, a survey of the theoretical component relating to the application of Benford's law in accounting. The data were collected from the Iberian Balance Sheet Analysis System database for 2019, 2020, and 2021. After analysing the data, significant deviations are observed in several digits, practically for all the compliance tests, both in the analysis of the first digit test and in the analysis of the first two digits test. The results therefore show signs of under-invoicing in 2020 by the analysed companies, which suffered, on average, a 79% reduction in turnover.

Keywords: COVID-19; Benford's law; under-invoicing; support; restaurant sector

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1 Introduction

According to the World Health Organization (WHO), the COVID-19 disease caused by severe acute respiratory syndrome coronavirus 2 (SARS-COV-2) was first detected in early December 2019 in Hubei province, in Wuhan, China. From 2020 onwards, COVID-19 spread globally, prompting the WHO to declare a worldwide pandemic on 12 March 2020. The impact of the pandemic has been felt health-wise and economy-wise (Ciotti et al., 2020).

The COVID-19 pandemic has had multidimensional effects, greatly impacting public health and economic activity (Strange, 2020). In Portugal, the government immediately adopted extraordinary measures to contain and mitigate the pandemic (Mamede et al., 2020), which damaged the economy (Strange, 2020). To select the companies that benefit from these measures, the government defined a set of criteria that they had to fulfil, namely, the reduction in turnover (VN).

This sought to detect signs of under-invoicing in companies to adjust their turnover to accomplish those criteria, maximising the support granted by the State. Indeed, it is expected that some companies could have reported lower than actual turnover to obtain that benefit. This study applied Benford's law to identify possible changes to the turnover reported by Portuguese restaurant companies in 2020, intending to maximise the benefits granted by the State in the context of the COVID-19 pandemic.

Benford's law is a useful analytical method for detecting the undervaluation or overvaluation of income (Green & Calderon, 1994). Benford's law was chosen because the VN item, due to its characteristics, fulfils the requirements to comply with this law (Durtschi et al., 2004).

The study is structured in five further sections. The next section presents the literature review to facilitate understanding of the topic in question. The third section describes the methodology used to answer the research question and the objectives of this study. The fourth

section presents the results obtained, and the fifth section discusses them. Finally, in the last section, the main conclusions of the study are presented, as well as the limitations and suggestions for future research.

2 Literature review

This section describes the impact that the COVID-19 pandemic has had worldwide and in the various sectors of activity in Portugal, presenting several policies adopted by the government for Portuguese companies, and the relevant theoretical components relating to the application of Benford's law in accounting.

The literature reveals a severe and sudden impact of the pandemic on sales revenues, in which the pandemic and the subsequent lockdown government measures severely affected the hospitality sector (e.g., hotels and restaurants) (Afonso & Calisto, 2023; Campobasso et al., 2023; Gomes et al., 2023; Janzen & Radulescu, 2022; Oikawa & Onishi, 2024). According to Janzen and Radulescu (2022), firms in the hospitality sector reported a significantly larger reduction in terms of year-on-year sales growth, and the gap concerning firms operating in other economic sectors intensified with increasing lockdown strictness. State support funding has been deployed to sustain the business, including cash transfers (subsidies), access to credit, deferral of payments, fiscal incentives, or wage subsidies to alleviate the negative repercussions of the lockdown (Campobasso et al., 2023; Janzen & Radulescu, 2022).

2.1 Impact of the COVID-19 pandemic

The COVID-19 pandemic is characterised by its worldwide impact and the fact that it had harmful repercussions on public health and most national economies (Strange, 2020). In Portugal, government measures to contain the pandemic were implemented, and, as a result, the economy deteriorated rapidly due to a sharp decline in employment, income, and consumption (Mamede et al., 2020).

2.1.1 COVID-19: Analysing the impact of the pandemic on companies

At the start of the COVID-19 pandemic (i.e., State of Emergency period), 83% of Portuguese companies remained in operation. Concerning the companies that were temporarily not in operation, they suffered a significant drop in all sectors of activity. However, this deterioration was most marked in the accommodation and restaurant sector (Banco de Portugal, 2020a). This sector, which normally accounts for 9% of Gross Value Added (GVA) and employment (Mamede et al., 2020), suffered an immediate negative impact on demand due to physical distancing measures imposed and the insecurity and uncertainty among the population.

Throughout 2020, on average, 80% of companies in the accommodation and restaurant sector reported a very negative impact on turnover, with April being the month in which 70% of companies recorded a decrease in turnover of more than 75%. These values subsequently decreased over the months, registering a significant reduction of 50% at the end of 2020. According to Sequeira et al. (2020), companies that remained temporarily closed had a greater tendency to exhibit a decrease in turnover of more than 75%, as was the case in the restaurant sector. As described below, the government launched a series of measures to support companies in mitigating the impact of the COVID-19 pandemic.

2.1.2 Support measures adopted by the government

This section briefly presents the measures to support the economy, liquidity, and cash flow of companies affected by the COVID-19 pandemic (economic support lines, bank moratoriums, exceptional and temporary tax compliance regime, and the APOIAR Programme).

According to Decree-Law no. 46-A/2020, of 30 July, the purpose of the extraordinary support for progressive recovery was to temporarily reduce the normal working period (NWP) in private and social sector companies in a "business crisis". To cover a greater number of companies benefiting from this support, the concept of a "business crisis" was changed, allowing companies that suffered a reduction of 25% or more in turnover to benefit from this measure, with a decrease in the NWP of 33%. As shown in Table 1 (in October, November, and December), the greater the drop in turnover, the greater the reduction in the NWP per employee (Decree-Law no. 90/2020, of 19 October).

To support liquidity and cash flow, the government instituted bank moratoriums to allow companies to postpone the payment of their liabilities, guaranteeing the continuity of company financing without any possible defaults (Decree-Law no. 10-J/2020, of 26 March). In terms of taxation, Decree-Law no. 103-A/2020, of 15 December, established the deferral of the payment of value added tax (VAT) in the first quarter of 2021 for companies under the monthly and quarterly regime that prove a drop of 25% or more in their turnover in 2020, to be paid in three or six payments, without interest rate.

Table 1. Limit for extraordinary support for progressive recovery.

Loss of turnover	≥25% to 40%	≥40% to 60%	≥60% to 75%	≥75%
Measure adopted by the State	NWP reduction of up to 33%	NWP reduction of up to 40%	NWP reduction of up to 60%	NWP reduction of up to 100%

Finally, the main objective of the APOIAR Programme was to create a liquidity incentive system for micro- and small enterprises (MSEs) and other companies with at least 250 employees and a turnover of no more than €50 million. This support covered the APOIAR.PT, APOIAR RESTAURAÇÃO, APOIAR RENDAS, and APOIAR+SIMPLES measures, with the support being cumulative with each other (Ministerial Order no. 271-A/2020, of 24 November, later amended by Ministerial Order no. 15-B/2021, of 15 January). Table 2 summarises the respective breaks in turnover, the amount of support to be received, and the effective dates of the support measures included in the APOIAR Programme.

The APOIAR.PT programme was changed in 2021, with Ministerial Order no. 15-B/2021, of 15 January, establishing new support limits for companies with a drop in turnover of more than 50%. As shown in Table 2, the amount allocated to each company was 20% of the drop in turnover when it had a decrease of at least equal to 25%.

Since the restaurant sector was considered one of the sectors most affected by the COVID-19 pandemic, the government adopted measures to support it. As shown in Table 2, the allocated funding was 20% of the average drop in turnover recorded over the two weekends with the most intense restrictions compared to the average for weekends from 1 January to 31 October 2020. On the other hand, the main purpose of the APOIAR RENDAS measure was to pay six monthly instalments to companies with a drop in turnover of 25% or more. According to Ministerial Order no. 15-B/2021, of 15 January, the rate of funding to be awarded varied according to the percentage of the drop in turnover: if it was between 25% and 40%, the amount of the monthly payment to be received was 30%, and if it was over 40%, the amount of the monthly payment was 50% (Table 3).

According to the Bank of Portugal (Banco de Portugal, 2020b), the restaurant segment was one of the sectors that most benefited from the support measures implemented by the government. Thus, most companies benefited from the simplified lay-off (65%) and the APOIAR Programme (58%). The APOIAR RENDAS programme and the bank moratoriums were among the support measures that received the least uptake from the respective sector.

It was concluded that most of the measures adopted by the government required a drop in turnover of 25% or more. This work aims to detect signs of under-invoicing in the companies under study to adjust their turnover in 2020 and maximise the support they received from the State in 2021. Notably, the analysed companies suffered a 79% reduction in turnover during this period, making

Table 2. Measures under the APOIAR Programme.

Measures	**Recipient**	**Loss of turnover**	**Support to be received**	**Effective date**
APOIAR.PT	Belong to the CAE in Annex A to Ministerial Order no. 69-A/2021, of 24 March	25% or more	20% of the amount of the decrease in the company's turnover	25 November 2020
APOIAR RESTAURAÇÃO	Belong to the CAE in Annex B to Ministerial Order no. 69-A/2021, of 24 March	20% or more (of the two weekends with the most intense restrictions)	20% of the amount of the decrease in the company's turnover	25 November 2020
APOIAR RENDAS	Belong to the CAE in Annex A to Ministerial Order no. 69-A/2021, of 24 March	25% or more	Table 3	16 January 2021

Table 3. Funding rate to be allocated—APOIAR RENDAS.

Loss of turnover	Amount receivable
≥25% and ≤40%	30% of the monthly rent up to a maximum of €1200 per month and per establishment
>40%	50% of the monthly rent up to a maximum of €2000 per month and per establishment

them susceptible to under-invoicing and allowing them to benefit from the State's incentives. Thus, the following research global hypothesis emerges: In 2020, restaurant companies with annual turnover declines show signs of under-invoicing.

H0: There are no signs of under-invoicing in restaurant companies in 2020.

H1: There are signs of under-invoicing in restaurant companies in 2020.

The next section presents some studies that use Benford's law on accounting data to detect signs of error or fraud.

2.2 Use of Benford's law with accounting data

The use of statistical methods in identifying and analysing earnings manipulation in finance and accounting is increasingly necessary nowadays, and the less complex, most popular, and still highly effective tests are the ones based on Benford's law (Slijepčevic & Blašković, 2014). Swanson et al. (2003) state that Benford's law is a practice most commonly used in the business sector, specifically in accounting. Several authors in the literature have analysed and examined the applicability of this law in specific areas, such as accounting, focusing on identifying the handling and alteration of results (Carslaw, 1988; Durtschi et al., 2004; Slijepčevic & Blašković, 2014; Thomas, 1989).

Carslaw (1988) applied Benford's law to accounting data from New Zealand companies with positive results. The author analysed the second digit of the profit category, detecting an excess of zeros in the second digit and, conversely, a scarcity of the digit 9. Carslaw (1988) concluded that companies' profits tended to be rounded up to higher values than expected to present a better image among stakeholders.

Like Carslaw (1988), Thomas (1989) discovered a similar pattern in the profits of American companies. Companies showing negative results had more nines in the second digit position and fewer zeros than expected. Niskanen and Keloharju (2000) observed that Finnish companies tended to adjust the second digit further to the left in the net profit category, as they observed an excess in the number 9, to make earnings appear sporadically larger. Nigrini (2005) used Benford's law to detect earnings manipulation at Enron (between 2001 and 2002), finding that most of the digits were above the reference points of the respective law, suggesting earnings manipulation in Enron's revenues and earnings per share.

Several ongoing scientific studies have analysed and applied Benford's law in accounting. Many studies use anomalies in the frequency of digits as evidence of the intentional actions (e.g., by managers) to manipulate the results, errors, or fraud of a given organisation. Aybars and Ataunal (2016) analysed the financial reports of companies listed on the Istanbul Stock Exchange between 2005 and 2015, detecting signs of manipulation due to various deviations in the total assets and net profit data. Like Aybars and Ataunal (2016), Gong et al. (2022) and Pupokusumo et al. (2022) used Benford's law on a set of companies to capture traces of fraud in financial data disclosed by companies in China and Brazil, respectively. Dharni (2020) proved the effectiveness of Benford's law in detecting *shell companies*, as he found a considerable distinction in the conformity of the distribution of Benford's law between real and fictitious companies in his study.

Nigrini and Mittermaier (1997) pointed out the logic of using Benford's law to help discover unusual patterns in economic/financial activities since it is very likely that an individual making fraudulent entries will use the same or identical numbers. However, Benford's law can never prove or disprove the presence of manipulated data; it only indicates that a given set of data does or does not follow the law, and if the analysed data do not closely follow the Benford curve, they should be considered for closer examination and scrutiny (Collins, 2017). According to Durtschi et al. (2004), Benford's law helps to detect suspicious data in financial statements so that these data can be analysed in detail and additional tests can be carried out, as Benford's analysis alone is not an infallible method of detecting fraud.

The following section describes the methodology used in this study.

3 Methodology

This section presents the research objective and hypothesis to be tested, the type of research used, and the units selected for analysis. Finally, the types of tests implemented for analysing the selected data are discussed.

3.1 Study objective and hypotheses from research

This study is centred on exploratory quantitative research, based on a prior literature review and formulated hypotheses to be tested later using statistical techniques. The VN of a group of Portuguese companies belonging to Economic Activity Code (CAE) 56 was analysed during the COVID-19 pandemic period, specifically between 2019 and 2021, with the aim to detect signs of under-invoicing through the application of Benford's law.

The main objective is to detect signs of anomalies, i.e., whether companies adjusted their turnover in 2020 to maximise the support they received from the State in 2021. To test the research global hypothesis, a significance level α of 5% (α=0.05) was established to test H0, measuring the risk of rejecting a true hypothesis. If H0 is rejected, it means that the VN values do not conform to Benford's law, i.e., suggesting potential evidence of data set manipulation. If H0 is not rejected, the VN values follow the expected values according to Benford's law, from which it can be concluded that the data analysed conform to the distribution of the law, and therefore, no evidence of under-invoicing.

3.2 Data collection and sample

The data were obtained from the Iberian Balance Sheet Analysis System (SABI) database. The data from Portuguese restaurant companies, CAE 56, were collected from 2019 to 2021 and processed and analysed in Microsoft Excel and SAS. The main purpose of choosing the period evaluated was to obtain data that would allow us to know the variations in companies' turnover before and during the COVID-19 pandemic.

The sample used consisted of 19,437 companies in the restaurant sector, the majority of which were micro-sized companies, a fact explained by the fact that this is the most prevalent type of company size in Portugal. It should be emphasised that, for a company to be included in the sample for this study, the following criteria were required:

- The company's headquarters and location must be in Portugal.
- It must operate in the restaurant business or a similar sector (CAE 56).
- The restaurant must have detailed financial information for 2019–2021.
- The restaurant must show a significant percentage reduction, 25% or more, in its turnover between 2019 and 2020.

3.3 Description of treatment method data

Firstly, we analysed the conformity of the data by calculating the frequency of the values found in the first digit, the second digit, and the first two digits, comparing them with the expected frequency of Benford's law.

The first digit and second digit tests are often used as high-quality preliminary and secondary tests, respectively (Drake & Nigrini, 2000). According to Nigrini and Mittermaier (1997), the first digit test is not intended to identify abnormal duplications or irregularities but only to check whether or not a data set follows the Benford distribution. The second digit test, on the other hand, signals rounding in values with an excess of the digits 0 and 5 than expected, compared to Benford's law (Drake & Nigrini, 2000). The first two digits test analyses and captures peaks (excess theoretical digits) in which the actual proportion, *Po*, is much higher than the expected proportion, *Pe* (Drake & Nigrini, 2000; Nigrini & Mittermaier, 1997). This test is essential for analysing a large data set and identifying duplicates and data errors in the event of poor compliance with Benford's law (Nigrini, 2011).

For each test, statistical tests known as the *Z*-statistic and the chi-square test (X^2) adopting Nigrini's model (2011) will be applied to analyse the conformity of the data.

The main purpose of the *Z*-statistic is to determine the statistical significance of the deviation between *Po* and *Pe*, signalling the digits that appear with greater or lesser regularity in a given position (calculation of the upper and lower limits in which the *Z*-statistic is equal to 1.96 with a significance level of 5%). Any digit that appears with greater or lesser regularity in a given position means that the calculation of the *Z*-statistic has been projected above or below the limit of Benford's law line (Nigrini, 2011). The *Z*-statistic is calculated as follows (Nigrini, 2011):

$$Z = \frac{|Po - Pe| - \frac{1}{2N}}{\sqrt{\frac{Pe(1-Pe)}{N}}} \quad (1)$$

where:
- *Pe* is the expected proportion for a given digit, according to the probability stipulated by Benford's law.
- *Po* is the proportion observed for a given digit.
- *N* is the number of registrations.

The main purpose of the X^2 test is to compare the relationship between *Po* and *Pe* by adding up the deviations in the position of a given digit in a sample. Like the *Z*-statistic, the X^2 test is also sensitive to the sample size (Nigrini, 2011).

The chi-square statistic for the X^2 test is calculated as follows (Nigrini, 2011):

$$X^2 = \sum_{i=1}^{K} \frac{(Po - Pe)^2}{Pe} \quad (2)$$

where:
- *Po* is the proportion observed for a given digit.
- *Pe* is the expected ratio for a given digit.
- *K* is the number of bins for a given digit.

With a significance level of 5%, the first and second digits are K=9 and K=10 (9 and 10 possible digits, respectively); thus, the number of degrees of freedom is eight and nine (k–1) for the X^2 test on the first and second digits. For the test on the first two digits, the degree of freedom is 89 (Nigrini, 2011). The critical values for the X^2 test are 15.51, 16.92, and 112.02, for the first digit, second digit, and first two digits tests, respectively (Nigrini, 2011). The chi-square statistic of the X^2 test is obtained by adding up all the parcels concerning the digits in a given sample. The null hypothesis is rejected when the value of X^2 exceeds the critical value defined for X^2 distribution.

The next sections present the main results and discussion.

4 Results

4.1 Sample characterisation

This section presents and analyses the results obtained from the data set mentioned in the previous section to verify that the sample is in accordance with Benford's law. Effectively, the sample presents a global reduction of 79% in turnover, on average, making them susceptible to under-invoicing and allowing them to benefit from the State's incentives previously mentioned. Three groups of companies are considered, taking into account the size, "Micro-enterprises", and the drop in turnover with "Variation 25% to 35%" and "Variation 35% to 75%".

Table 4 shows the minimum (Min), maximum (Max), average ($\bar{x}$), standard deviation (s), and number of observations (n) for the total and groups of companies for the

Table 4. Descriptive statistics.

Group	*n*	Statistics	VN	TA	SE
Total	19,437	Min	€42	€79	€2
		Max	€102,839,963	€190,580,920	€6,148,155
		$\bar{x}$	€259,074	€269,928	€26,997
		s	€1,723,541	€84	€70,473
Variation 25% to 35%	5,356	Min	€900	€320	€44
		Max	€85,942,653	€146,861,014	€2,065,550
		$\bar{x}$	€299,299	€267,355	€21,054
		s	€1,385,987	€6,220,523	€80,215
Variation 35% to 75%	7,744	Min	€128	€84	€13
		Max	€35,079,616	€46,613,670	€2,641,701
		$\bar{x}$	€213,915	€283,069	€30,914
		s	€738,058	€4,964,952	€63,895
Micro-enterprises	19,090	Min	€42	€79	€2
		Max	€1,995,478	€1,999,423	€352,160
		$\bar{x}$	€179,797	€168,190	€19,950
		s	€234,318	€3,038,999	€37,210

following values: VN, total assets (TA), and operating subsidy (SE) for 2020.

As shown in Table 4, in the Total group, which includes all companies under study, the maximum value for VN is €102,839,963 and that for TA is €190,580,920. In the "Variation 25% to 35%" group, it can be seen that VN has a $\bar{x}$ of €299,299. TA shows values between €320 and €146,861,014. The third group, "Variation 35% to 75%", presents all the companies that showed a percentage reduction in their turnover between 35% and 75% in 2020, representing around 40% of the sample. Looking at these values of the turnover and TA, it can be seen that these are smaller companies than those in the previous division; i.e., the smaller companies recorded a greater reduction in turnover in 2020. The "Micro-enterprises" section shows all the companies with a turnover and TA of €2,000,000 or less. Thus, it can be concluded that most of the companies under study are predominantly micro-sized, and as a result, it was impossible to analyse the data set that belonged to other dimensions due to the limited number of companies available for analysis using Benford's law.

4.2 Benford's law—Analysing turnover compliance

According to Benford's law, Table 5 shows the results obtained by applying the *Z*-statistic and the X^2 test to the first digit in the various groups of the sample under study.

By applying Benford's law, all the groups have anomalies, except for "Variation 25% to 35%" which has a X^2 equal to 7.9 (below its critical limit) and a *Z*-statistic lower than its critical value of 1.96, in all digits from 1 to 9.

The groups "Variation 35% to 75%" and "Micro-enterprises" have a X^2 value equal to 16.9 and 53.9, respectively, rejecting H0. Thus, these groups fail to comply with Benford's law, presenting a value of X^2 higher than its critical value, mainly the "Micro-enterprises" group.

The *Z*-statistic shows that digits 1 to 4 and 9 have a value above their critical threshold in the "All" and "Micro-enterprises" groups. The "Variation 35% to 75%" group only shows a value above its critical threshold in individual digits 2 and 4. It can be seen that the *Po* is higher than the *Pe* (excess) in individual digits 1 and 9 and, on the other hand, there is a shortage in digits 2, 3, and 4 for the "All" and "Micro-enterprises" groups. The "Variation 35% to 75%" section has a *Po* lower than the *Pe* in digits 2 and 4.

In this sense, it can be concluded that H0 is not rejected for the "Variation 25% to 35%" group, since there are no statistically significant differences between *Po* and *Pe*, at a significance level of 5%, according to Benford's law. H0 is rejected for the remaining groups, as their data do not conform to Benford's law, considering the values obtained by the *Z*-statistic and the X^2 test.

Compared to other studies similar to this one, there is a greater deviation between *Po* and *Pe* in the first digit test, which may suggest a potential duplication of numbers.

Table 5. First digit test.

First digit	All			Variation 25% to 35%			Variation 35% to 75%			Micro-enterprises		
	Po	*Pe*	*Z*	*Po*	*Pe*	*Z*	*Po*	*Pe*	*Z*	*Po*	*Pe*	*Z*
1	6079	5851	**3.6**	1097	1064	1.2	2400	2331	1.7	6003	5747	**4.0**
2	3313	3423	**2.1**	615	623	0.3	1288	1364	**2.3**	3179	3362	**3.5**
3	2311	2428	**2.5**	424	442	0.9	958	968	0.3	2256	2385	**2.8**
4	1749	1884	**3.3**	322	343	1.2	691	750	**2.3**	1718	1850	**3.2**
5	1525	1539	0.4	299	280	1.2	616	613	0.1	1511	1512	0.0
6	1285	1301	0.5	221	237	1.1	512	518	0.3	1274	1278	0.1
7	1160	1127	1.0	222	205	1.2	468	449	0.9	1152	1107	1.4
8	1028	994	1.1	168	181	1.0	430	396	1.7	1019	977	1.4
9	987	889	**3.4**	168	162	0.5	381	354	1.4	978	874	**3.6**
X^2			5.13			7.9			**16.8**			**53.9**

Z-statistic		X^2	
Critical value	1.96	Critical value	15.51

Next, we present the results obtained by applying the second digit test, illustrated in Table 6, for the groups analysed.

The X^2 test does not exceed the stipulated critical value of 16.02, for a significance level of 5%, for any of the groups analysed. There is only one violation of the critical value of *Z* in the "All" section for digits 0 and 9. Digit 0 is in excess, whereas digit 9 has *Po*<*Pe* (scarcity). In the other groups, the *Z*-statistic complies with Benford's law, since the value of *Z* obtained for digits 0 to 9 does not exceed its critical limit. However, the "Micro-enterprises" section shows a *Z* value close to its critical limit for the individual digits 0 and 9.

For the second digit test, it can be concluded that H0 is rejected only for the "All" group since there are statistically significant differences between *Po* and *Pe*, at a significance level of 5%, according to Benford's law, considering the values obtained by the *Z*-statistic. According to Benford's law, H0 is not rejected for the remaining groups, as there are no statistically significant differences between *Po* and *Pe* at a 5% significance level.

Table 7 shows the results obtained by applying the tests to the first two digits. Table 7 only illustrates the digits with the most significant deviation between *Po* and *Pe*.

Analysing the X^2 test, it can be seen that all the groups have a statistic lower than their critical value, except for the "All" and "Micro-enterprises" groups. These groups are incompatible for the test of the first two digits, since the statistics obtained (120 and 131) are higher than the critical value.

If the digits are evaluated individually, there is a violation of the critical value of *Z*-statistic in all the groups analysed, with 14 digits in which the value obtained exceeded the threshold (1.96), for a significance level of 5%. It can be seen that individual digits 10, 11, 44, 49, 76, and 98 show a significant deviation between *Po* and *Pe* for both the "All" and "Micro-enterprises" groups. The remaining groups, "Variation 25% to 35%" and "Variation 35% to 75%", show non-compliance with Benford's law, according to the value obtained for the Z-statistic, in the digits 50, 57, 76, 79, and 98 and 11, 24, 44, 47, and 88, respectively. The digits 10, 11, 57, 76, 79, 88, 90, and 98 have a *Po* greater than *Pe* (excess), according to Benford's law, while there is a scarcity in the remaining digits (24, 29, 44, 47, 49, 50, and 88), with *Po*<*Pe*.

In this sense, we conclude that H0 is rejected for all groups, since they show statistically significant differences between *Po* and *Pe*, for a significance level of 5%, according to Benford's law, considering the values obtained by the *Z*-statistic and the X^2 test.

The results presented, especially in this last test, show signs of under-invoicing in companies in the restaurant sector in 2020, allowing them to benefit from the incentives provided by the State in 2021.

Table 6. Second digit test.

Second digit	All			Variation 25% to 35%			Variation 35% to 75%			Micro-enterprises		
	Po	***Pe***	***Z***	***Po***	***Pe***	***Z***	***Po***	***Pe***	***Z***	***Po***	***Pe***	***Z***
0	2415	2326	**2.0**	432	423	0.5	974	927	1.7	2368	2285	1.9
1	2270	2214	1.3	421	403	1.0	926	882	1.6	2222	2174	1.1
2	2108	2115	0.2	387	385	0.1	826	843	0.6	2078	2077	0.0
3	1972	2028	1.3	343	369	1.4	782	808	1.0	1937	1992	1.3
4	1935	1950	0.4	353	355	0.1	746	777	1.2	1895	1915	0.5
5	1908	1879	0.7	342	342	0.0	776	749	1.1	1873	1846	0.7
6	1779	1815	0.9	359	330	1.7	698	723	1.0	1753	1783	0.7
7	1759	1756	0.1	305	319	0.8	705	700	0.2	1726	1725	0.0
8	1719	1702	0.4	321	310	0.7	681	678	0.1	1688	1672	0.4
9	1572	1652	**2.1**	273	301	1.7	630	658	1.1	1550	1623	1.9
			11.7			8.9			10.1			10.1

Z-statistic		X^2	
Critical value	1.96	Critical value	16.92

Table 7. The first two digits test (some results).

First two digits	All			Variation 25% to 35%			Variation 35% to 75%			Micro-enterprises		
	Po	*Pe*	*Z*	*Po*	*Pe*	*Z*	*Po*	*Pe*	*Z*	*Po*	*Pe*	*Z*
10	861	805	**2.03**	166	146	1.66	354	321	1.91	855	790	**2.35**
11	808	734	**2.76**	142	134	0.74	352	293	**3.54**	796	721	**2.83**
22	343	375	1.68	73	68	0.58	127	149	1.86	335	369	1.76
24	316	345	1.55	58	63	0.60	112	137	**2.18**	299	338	**2.16**
29	260	286	1.56	45	52	0.99	101	114	1.23	248	281	**1.99**
43	168	194	1.88	33	35	0.39	67	77	1.18	164	191	1.94
44	157	190	**2.39**	28	35	1.11	57	76	**2.15**	154	186	**2.38**
47	161	178	1.26	36	32	0.65	52	71	**2.25**	158	175	1.26
49	142	171	**2.20**	24	31	1.27	68	68	0.01	142	167	**1.98**
50	156	167	0.87	18	31	**2.26**	70	66	0.42	154	164	0.80
57	157	147	0.84	37	27	**2.00**	68	58	1.25	157	144	1.07
76	131	110	**1.97**	32	20	**2.67**	45	44	0.16	131	108	**2.18**
79	106	106	0.02	29	19	**2.21**	36	42	0.97	106	104	0.17
80	112	105	0.70	15	19	0.94	52	42	1.59	112	103	0.89
88	108	95	1.29	11	17	1.53	51	38	**2.11**	107	94	1.38
90	113	93	**2.05**	14	17	0.72	47	37	1.62	110	92	1.93
96	104	87	1.77	22	16	1.53	42	35	1.21	104	86	1.96
98	122	86	**3.93**	27	16	**2.90**	38	34	0.66	119	84	**3.80**
X^2			**120**			98			106			**131**

Z-statistic		X^2	
Critical value	1.96	Critical value	112.3

5 Discussion

As defined, the main objective of this work is to identify signs of under-invoicing, not evidence of under-invoicing. Remember that one of the conditions for a company to be included in the sample was to have recorded a 25% or more decrease in turnover between 2019 and 2020. The sample presents a global reduction, on average, of 79%.

The statistically significant deviations were found between *Po* and *Pe*, using the X^2 test when applying the first digit and the first two digits tests, for the groups "Variation 35% to 75%" and "Micro-enterprises". Specifically, the *Z*-statistic of the first digit test enabled the identification of statistically significant deviations in these groups and the whole sample. Additionally, the *Z*-statistic of the second digit test only detected statistically significant differences for the entire sample, for digits 0 and 9. Furthermore, the *Z*-statistic of the first two digits indicates the violation of the critical value of the *Z*-statistic in all the groups analysed. These results may be associated with possible rounding or duplication of data with the aim of these companies presenting a different percentage reduction in turnover than expected by under-invoicing, and thus maximising the benefits of the subsidies that the State presented during the pandemic period.

The signs of under-invoicing do not differ so much according to the decrease in turnover but according to the size of the companies. The smallest companies show the greatest signs of under-invoicing.

As observed in other studies similar to this one, it is expected that the test of the first two digits would exhibit a higher degree of deviation with Benford's law, because it is a test that more effectively detects certain anomalies that other tests may not identify.

Based on the sample results from all companies that complied with Benford's law, further analysis is conducted to consider the location variable using NUTII, enabling analysis of the under-invoicing effect of restaurants

in more touristic regions. Of the 19,437 companies, it was only possible to identify the geographical location of 16,728. Thus, a variable was constructed to determine whether a particular company in 2020 did not follow the distribution of Benford's law for the test conducted to the first two digits. The analysis revealed an average of 11.48% of companies in the sample whose turnover in 2020 did not follow Benford's law across all locations, ranging from a minimum of 9.96% in the Autonomous Region of Madeira to a maximum of 12.75% in the Autonomous Region of Azores (10.91% in Alentejo; 11.50% in the North, Centre, and Lisbon; and 12.1% in Algarve). Despite the difference in frequency of companies in each region whose turnover does not follow Benford's law, there are no statistically significant differences among them (*P*-value = 88.8%, Kruskal–Wallis test).

6 Conclusions

The main objective of this work is to detect signs of under-invoicing by applying Benford's law to the restaurant sector (CAE 56) during the COVID-19 pandemic in 2020, in which the analysed companies suffered a 79% reduction in turnover, on average. It should be emphasised that this method does not reliably ensure the occurrence of under-invoicing or errors, since the law simply detects the presence of statistical significance deviations between the observed data set and the expected data set.

Benford's law was applied to the VN category in several groups of the analysed companies—"All", "Variation 25% to 35%", "Variation 35% to 75%", and "Micro-enterprises"—to compare similarities and differences between *Po* and *Pe* and to find out whether the data analysed are distributed according to Benford's law or whether there are significant irregularities and deviations.

The analysis shows significant deviations in several digits for practically all the compliance tests, both in the first digit test and in the first two digits test, implying signs of under-invoicing in 2020 for some companies. The smallest companies show the greatest signs of under-invoicing.

The analysis revealed an average of 11.48% of companies in the sample whose turnover in 2020 did not follow Benford's law across all locations (using NUTII), ranging from a minimum of 9.96% in the Autonomous Region of Madeira to a maximum of 12.75% in the Autonomous Region of Azores. Despite the difference in frequency of companies in each region whose turnover does not follow Benford's law, there are no statistically significant differences among them.

For future research, it would be interesting to cross-check the results obtained with other methodologies commonly found in the literature. These include models based on accruals (discretionary and non-discretionary), the real sales manipulation model, and Burgstahler and Dichev's (1997) distributional model. The objective is to assess the potential for under-invoicing and further consolidate the use of Benford's law to detect accounting fraud. This would be valuable for competent authorities, professionals in the field, and other users of companies' financial information. It would be interesting to conduct a study similar to this one for the year 2021, a period during which companies were also affected by the COVID-19 pandemic. The primary objective is to compare the results of this study obtained by applying Benford's law with those obtained for 2021 and to determine whether the companies under study continue to exhibit significant deviations.

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