sciendo | NS global economics | ECONOMICS
Innovative and Economics Research Journal
Volume 13, No. 1, 2025.
www.economicsrs.com

A HYPERGAME MODEL OF CONFLICT CUSP HELIX FOR NONCOOPERATIVE BOUNDEDLY RATIONAL MULTI-POLAR ACTORS USING DEEP ADVERSARIAL REINFORCEMENT LEARNING AND MISPERCEPTION INFORMATION STRATEGIES

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Received: 28.10.2024. | Sent to review: 18.11.2024. | Accepted: 25.11.2024.

Original article

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JEL Classification: D74, C63, C73, C32, C45, H56

Doi: 10.2478/eoik-2025-0010

UDK: 37.018.43:004.738.5

ABSTRACT

In the aftermath of the Cold War era, the world shifted from a tale of two competing superpowers, to a highly complex and multi-polar narrative. In a new world order, geopolitical relations, diplomacy, power plays and conflicts among countries became so perplex that any attempt to emulate realistic scenarios, conduct robust simulations, all the more so deduct foreseeable outcomes, would be nothing short of tenuous, if not practically infeasible. Besides, unlike physical systems for which universal uniformities factually and fundamentally exist, social systems do not obey implicit and exact "natural-laws-of-physics". And while a plethora of modeling approaches have been deployed hitherto to tackle with the intrinsic complexity of warfare dynamics, however the required interdisciplinary synthesis has not been explored in the literature. In a first attempt to attain this fusion of cutting-edge methodologies, I propose a novel hybrid approach to conflict analysis.

Keywords: *warfare, multi-polar world, deep reinforcement learning, catastrophe theory, bounded rationality, Helix conflict, hypergame theory, complex systems, chaos*

1. INTRODUCTION

In 2022 a new "curtain" emerged similarly to the Iron Curtain after the WWII, and perhaps even more threatening: a "nuclear curtain". The world entered a new "Cold War" era after Russia's invasion in Ukraine, and nobody can predict how long it will take to overcome. Meanwhile, in 2023 another warfare front emerged following Hamas's attack on Israel. Regardless of any

* This work is based on the seminal presentation delivered by the author at the "*2nd International Conference on Social Capital, Economic and Political Strength of Countries in the Middle East*" that was jointly organized by the "*Institute of Economics, Society and Peace in the Middle East"* and *"Western Galilee Academic College*" in June 21-23, 2022at the city of Acre, Israel. I would like to thank the Director of "Oikos" Institute - Research Centre, Professor Dr. Zoran Mastilo for his valuable contribution. Moreover, I cordially thank the organizing committee of the "*2nd International Conference on Social Capital, Economic and Political Strength of Countries in the Middle East*" and particularly Prof. Gil Cohen, Dr. Carmit Gal, Prof. Hagit Turjeman, Prof. Michael Laskier and Dr. Haim Sperber, as well as the conference participants for their comments and suggestions during the presentation.

warfare discontinuation or standstill occurring in the aftermath of the two aforementioned conflicts, the initiation of escalation is bound to inflict a multi-polar geopolitical equilibrium state for decades ahead. Under a more scientific systemic context, and as opposed to the Cold War era wherein the two major superpowers confronted one another, in a multi-polar world with three or more actors 'entering' the global sphere, episodes of low scale or very intense geopolitical and economic oscillations could evolve in an uncontrollable fashion; yet not due to exogenous shocks but mainly due to endogenous perturbations. In this work, I utilize an advanced complex systems approach. Beyond the classical "game-theoretic" Nash context in geopolitics, scientific approaches such as the Helix model can provide invaluable insight. However, even complex systems modelling, under many spatially and temporally varying variables and uncertainty parameters, could only be marginally reliable.

Decision theory and more importantly hypergame theory, can be used to model war conflicts. Specifically, when very little is known about the actors involved (opponents), game theory can be used for adversarial reasoning. Moreover, decision theory in combination with game theory may render a better choice if the opponents are well known, which is often the case in complete information games. If one or more actors, a.k.a global "agents", are 'playing' different games considering that they are not fully aware of the nature of the game, then a hypergame approach can be used. The opponents in such framework, could be rational, irrational, or more interestingly – and realistically so - "boundedly rational".

According to Gilboa et al. (2010), the informal definition of 'objective rationality' revolves around the ability to convince others. Unfortunately, in many decision problems under uncertainty, a preference relation that can be interpreted as 'objectively rational' would fail to be complete. "Subjective rationality' does not require that the decision maker convince others being right, only that others will not be able to convince this decision maker that is wrong. Bounded rationality does not mean irrationality, since players want to make rational decisions but cannot always do so because fully rationality requires unlimited cognitive capabilities. The "bounded rationality" approach allows each player to make decisions based on its own perceived state of the game or environment, leading to multiple players having different perceptions of the conflict interaction. In noncooperative games or hypergames, actors (players) may derail from strategic incentives to cooperate, i.e., sovereign states are not behaving as rational individuals.

Furthermore, recent novel developments in Artificial Intelligence, Big-data, Quantum computing, Genetic algorithms, Sentiment analysis and Machine Learning, could be deployed in conjunction with hypergames, to embed features of realistic scenarios and emulations. Reinforcement Learning is a model-free computational technique aiming at the automation of goal-directed decision-making in a dynamic environment (Kaelbling et al., 1996). The agent uses a signal to determine an optimal policy which results in maximum reward, total utility or minimum errors. The learning mechanism particularly of boundedly rational agents (players) "decodes" the knowledge-acquisition strategy and their decision-making process via rules-of-thumb for their decisions under uncertainty shocks. Integrating AI-driven fuzzy logic, can further incorporate heterogeneous agent beliefs and perceptions as represented by "Dempster–Shafer belief" functions, as well as "inference rules" with time-varying parameters estimated by means of adaptive training, for each actor (Bekiros, 2010).

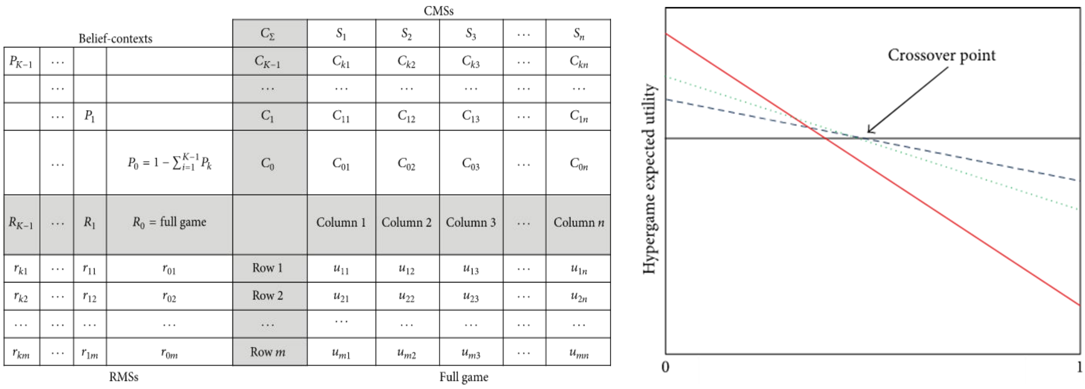
Human-driven social, economic and geopolitical systems are extremely hard to simulate efficiently. These systems are continuously temporally and spatially adaptable, auto-adjustable, often exhibiting chaotic behavior and highly volatile with modifying initial conditions, rendering long- or even mid-term prediction almost infeasible. In the present work, I develop a hybrid dynamic algorithmic hypergame approach utilising a chaotic cusp-like Helix model, applied to military conflicts with deception and (mis)perception information strategies. It allows for boundedly rational adversarial reasoning via integrating evolutionary AI/ML-driven ensemble learning modelling. I run simulation scenarios exploring various "resolution" states of a geopolitical warfare, such as the Israel-Hamas or the Russia-Ukraine conflict. When I incorporate the hypothesis of a two-pole superpower world (like in the Cold War era), the results seem to lead consistently to an "optimal" albeit dystopic resolution scenario, namely one of the opponents "sacrificing" to end the warfare. This is widely known as the "volunteer's dilemma" in game theory. War stops, yet inflicting long-term irreversible consequences for the opponent sacrificed. Even in case I utterly exclude this option (volunteer's dilemma) in calibrating my strategic preference modelling, and after running a multitude of simulation variations, the outcome remains rather unchanged. When I un-restrict the system vis-a-vis this constraint, via highly penalising it with a hyperparameter set, the system resolves once again into an irrational last-resort "volunteer's dilemma" adversarial conditionality, which exhibits the highest combinatorial utility of the fully-fledged hypergame.

Nevertheless, in a modest attempt to emulate a "multi-polar world" by involving into the hypergame just one additional third actor/power, the system dynamics become unprecedentedly modified, highly nonlinear and chaotic, forcefully perturbating towards other equilibrium states. Interestingly, almost all scenarios do not evolve towards a global optimal equilibrium state. The most probable outcome entails observing endogenously generated cusp-like partial equilibria with inherently unpredictable oscillations that become even more "violent" and unforeseeable in the presence of high levels of "noise". In these settings, "noise" could be perceived as being enforced exogenously through stochastic political, military or economic uncertainties, triggered by variant factors amidst the unstable international environment. This entirely different dynamical behavior pertains even as time passes, the system continuously updates and the next stages of the hypergame model are temporally incorporated. In the following sections, I briefly describe key-terms for each approach embedded in the system, I introduce the notation, and lastly I provide some useful inference conclusions. I indicate that the proposed model, with only minor modifications contextually appropriate to fit the corresponding application, can be utilised in other fields, such as in diplomatic affairs, business strategies, bargaining, multilateral negotiations and numerous other.

2. HYPERGAME THEORY AND CONFLICT ANALYSIS

Hypergame theory extends game theory by allowing unbalanced modeling which represents the differences in each player's information, beliefs, and understating of the game. Kovach et al. (2015) provide an excellent overview of hypergame theory. The unbalanced game model allows for a diverse actor's view, whilst an overlap could be also accommodated when there is common knowledge. The outcome or solution to the hypergame is dependent on each agent's perception, including how each one views the game and how the actor believes the opponent is viewing the game. In a hypergame, each player may i) possess a false or misled understanding of the preferences of the other players, ii) have incorrect or incomplete comprehension of the actions available to the other actors, iii) not have awareness of all the agents in a game or iv) have any combination of the above: faulty, incorrect, incomplete, or misled interpretations. A player's choice of actions (decisions) reflects its own understanding of the game outcomes; the actor chooses actions based on the way they perceive "reality", albeit it may not be the true state of reality.

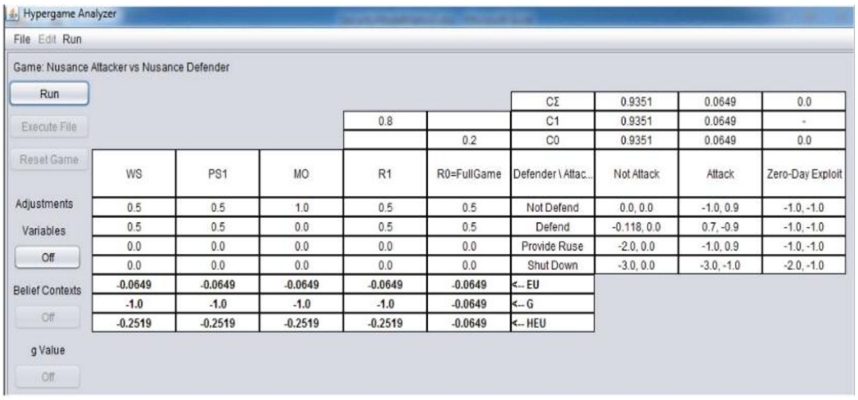
Figure 1. (Kovach et al, 2015)

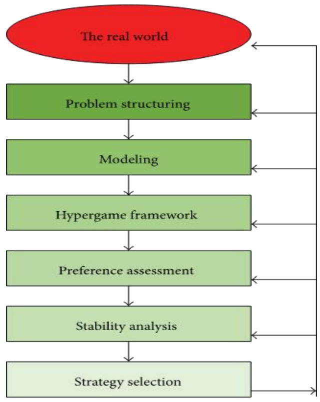


Vane (2000) offered a different approach by allowing the incorporation of a player's beliefs into the opponent's possible actions and provided a graphic representation that is evocative of the normal strategic form used in standard game theory. This model is referred to as Hypergame Normal Form (HNF). Risk assessment is built into the hypergame via a method termed "quantified outguessing", i.e., introducing the "fear" of the actor that can be outmaneuvered. Huxham and Bennett (1983) built up a structured overview of hypergames, advocating that a structured overview will often be too complex to embed into a formal hypergame model. It is necessary to abstract farther, making simplifications by asking specific questions, e.g., a) how do different problem aspects relate? b) can simplifications be made to the complex model while retaining its basic overview? and c) which participants are most important or influential? Moreover, Hipel and Dagnino (1988) presented an approach with two or more decision-makers, whereby one or more of agents have misperceptions. The algorithm is called the hypergame cooperative conflict analysis system (HCCAS). HCCAS unifies work in hypergame theory and conflict analysis.

Fraser and Hipel (1980) developed Conflict Analysis Program (CAP) based on the IEE/WINFORMS joint Program for Capital Science, using MATLAB or Mathematica. Their software can calculate the utility functions from mathematical equations and run multiple iterations and update variables as well as player beliefs between iterations. In case this software is utilized for each game, it'd lead to the entire model being built from scratch. Gambit software is used for finite, non-cooperative games, yet its main disadvantage is the lack of support for complex hypergames. HYPANT developed by Brumley (2003) allows the hypergame data to be saved and restored, supports sub-games based on the player's perceptions, but lacks functional utility values. Gibson (2013) created the HNF analysis tool (HAT) in Java and XML. HNF allows multiple game iterations to be run, supporting static or a random strategy selection, while updating variables and beliefs between hypergame iterations.

Figure 2. (Kovach et al, 2015)

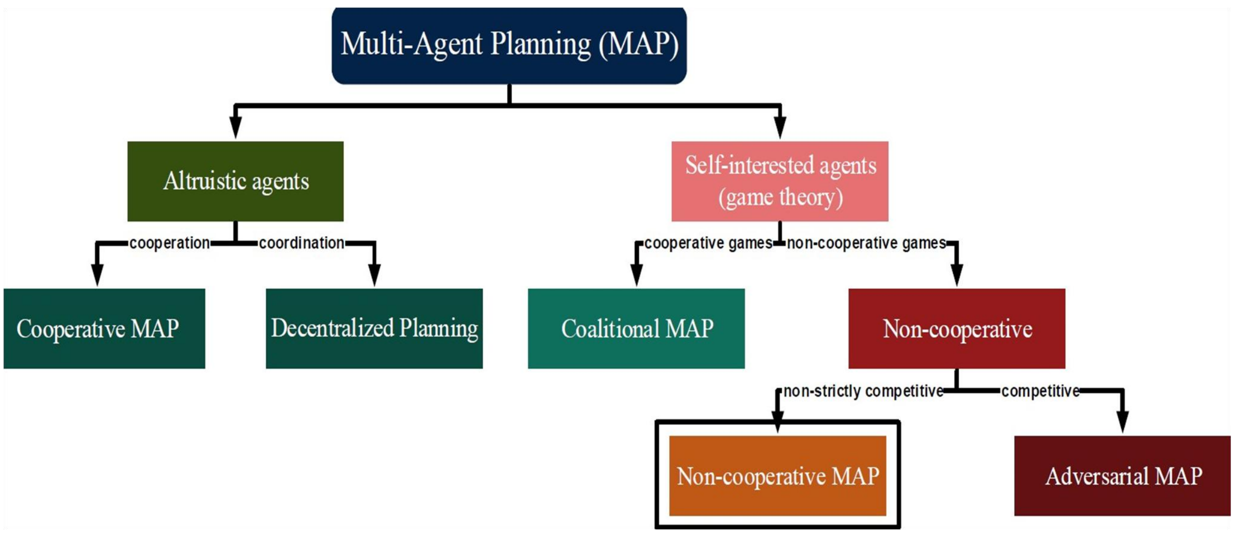




Bennett and Dando (1979) first applied hypergames to a real-world scenario via the analysis of the Fall of France during WWII. Wright et al. (1980) presented a more complex hypergame for the nationalization of the Suez Canal in the 1950s, wherein the modeled conflict could lead to strategies changing over time, yet one at each stage. Rott (2011) examined the Falkland/Malvinas conflict between Britain and Argentina in 1982, and showed how misperceptions dictated an outcome that was unexpected by all sides. This model integrated three different hypergames, simulated at multiple time points, yet it didn't factually contain temporal aspects. Fraser et al. (1983) applied five different conflict models to foresee the result of a possible nuclear confrontation between the USA and Soviet Union during the Cold-War era. Said and Hartley (1982) used hypergame theory to analyze the 1973 Middle East War. They concluded that each player behaved in a rational manner, yet within their own perceptual beliefs. The latter could be broadly considered as a first attempt to model boundedly rational or irrational actors.

Hypergames can also utilize a plethora of advanced approaches such as nonlinear dynamics, fuzzy logic, Bayesian or Agent-based modeling, Artificial Intelligence, Machine Learning etc. More computational hypergame architectures can be seen in Figure 3. In my work I attempt for the first time in the literature, to combine HAT with Helix approaches, catastrophe theory, stochastic modeling, chaotic dynamics, fuzzy logic, Reinforcement learning, and more importantly extend the hypergame to multiple players.

Figure 3. (Kovach et al, 2015)



3. BELIEF FUNCTIONS, IMPOSSIBILITY THEOREM AND NON-DICTATORIAL UNCERTAINTIES

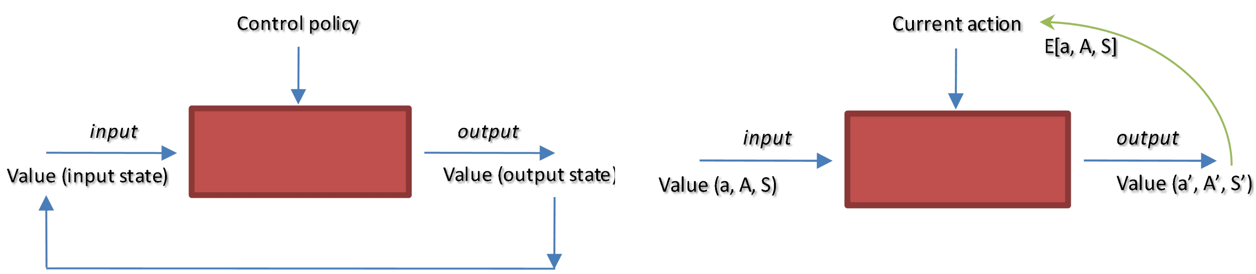
Possibility theory deals with certain types of uncertainty and could express a plethora of qualitative traits within a hypergame. Originating from fuzzy logic, the “Dempster–Shafer belief functions” can be incorporated. They are also called possibility functions ranging within the [0,1] interval, and expressing marginal membership for each qualitative trait e.g., “very”, “little”, “high, “low”, “crucial”, “neutral”, etc. The transformation of numeric input values into belief/perception/preferences, is called fuzzification (Bekiros, 2010). A valuable theorem in this context is the so-called impossibility theorem (Nehring, 2006). It poses that: “Impossibility renders any non-dictatorial mechanism for aggregating individual beliefs to inform collective decisions risks selecting outcomes that no one favors”. If future beliefs/perceptions are uncertain and/or change over time, then any non-dictatorial mechanism for decisions on present and future preferences and beliefs, may choose an action that no one favors. Yet, impossibility theorems cannot be modelled within a mechanism-design game theory framework.

Furthermore, in noncooperative games, actors may derail from strategic incentives to cooperate or not. Realistically, sovereign states are not behaving as rational individuals. In these games, utilities can be modelled to change over time, to embed temporal changes. Even in complex game theory, uncertainties about the emergence of dangerous strategies may turn a coordination game into the so-called “prisoner’s dilemma” (Barrett and Dannenberg, 2014). It assumes that all players (countries) are rational with coherent preferences. However, it ignores internal disagreements, non-rational drivers of perceptions, priorities, and preferences, and ergo traits that are factually fundamental in real-world applications. In realistic emulations, perceptions of fairness and morality and/or coalitions amongst countries, are easily manipulated and not necessarily coherent and sincere. This is called “moral dumbfounding”, and it was initially coined by Haidt et al. (2000). Expectations could be estimated via well-known statistical and econometric models, or more recently through the use of heuristic behavioral rules as presented by Hommes (2013). Overall, the terms beliefs, expectations, preferences, perceptions in social sciences literature tend to be used identically and most of the time rightfully so. In my work I introduce a different approach to modeling beliefs (and preferences), expectations and (mis)perceptions, separately. I consider perception as the vector sum of beliefs and expectations (after estimation) in a complex number plane, whereby perception is the complex number (vector) comprising the real-axis belief value plus the expectation value which lays on the imaginary y-axis. Misperception (deception) could be formalized as the complex conjugate, considering that beliefs are usually ‘fixed’, while expectations could fluctuate for the same game (and actor) between optimistic (positive) and pessimistic (negative)as depicted on the imaginary axis. However, this analysis goes beyond the scope of the current paper, and for the sake of simplicity I will follow the norm of terms’ tautology.

4. CHAOTIC DYNAMICS AND STOCHASTICITY

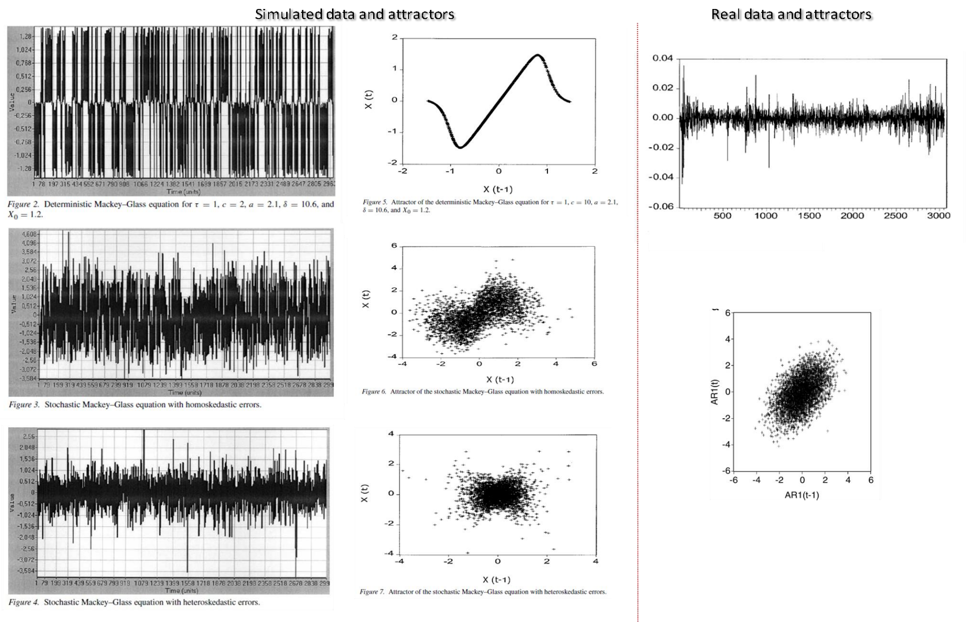
Stochasticity in social sciences, as in classical physics, can be considered as an indication of “ignorance”. An eloquent paradigm stems from the coin experiment. In particular, classical probability theory dictates that if we flip a coin, we have 50% probability of getting head or tails. The same applies when we add an error (stochastic) component in a hypergame for war-conflict modeling. However, chaos theorists proved that if we know the shape and weight of the coin, the strength of the tosser, the atmospheric condition of the room in which the coin is tossed, the distance of the tosser's hand from the ground, and many other environment (state) variables/parameters, we could predict with certainty whether it would be head or tails. Therefore, stochasticity as provisioned in probability theory, represents our ignorance of accurate state measurements. Only in quantum mechanics, probability is considered inherent in micro-cosmos, whereby even in case of perfect knowledge of the initial conditions and state variables, we can never make accurate forecasts. Unlike natural sciences, whereby we don't cope with non-causal systems, in social sciences we always do. Non-causality boils down to expectations' formulation and their integration into the complex system. Incorporating expectations can only be systemically and mathematically feasible, via a feedback mechanism as encountered in control engineering theory. Yet the "stretch" involves inducing a pattern of leads (representing expectation formation), instead of commonly utilizing the lags (past and present states) in natural systems.

Figure 4.



In nature, when a physical system is forced to move by a "force", it perturbs and could reach an equilibrium state or not. It may fluctuate between partial equilibria or in extreme and complex cases it'd display jumps and unresolved disequilibria. Commonly, it is considered that extreme jumps and non-equilibrium states are not inherent nor endogenously caused in systems; they are only observed when a "shock" is enforced upon a system that is exogenous and stochastic. Albeit counterintuitive at first, however alternating states with cusps of dis-equilibria and temporal stable plateaus, are endogenously generated by simple deterministic maps (functions). In nonlinear and chaotic systems, a specific number of "modes" is created after initial perturbation. For example, we could assume the movement/vortex of a flow field. The corresponding paradigm could be represented via a geopolitical social system (fluid), in that when many modes (actors) are enforced then the "flow" becomes irregular. Specifically, it is proven that when three or more modes (also referred to as frequencies/oscillations) unfold within a system, this will become endogenously chaotic, as described by the Landau-Hopf turbulence dynamics (Landau, 1944; Hopf, 1948). The irregularity is generated not by exogenous (random/stochastic) influences, but inherently from endogenously embedded parameter perturbations. Most frequently, these systems are represented by deterministic functions or partial differential equations with highly sensitive parameters. In physical systems when exogenous shocks impact a complex deterministic system, we can observe turbulence in the form of periodic or quasi-periodic attractors. However, according to Takens's theorem (1981), in some cases we may observe chaotic (strange) attractors with continuous spectra and period doubling (Feigenbaum effect). When we do not know the exact equations but we acquire variable observations (via measurement) of samples entailing vast length and adequate measurement accuracy, we could closely reconstruct the system dynamics topologically (Figure 5).

Figure 5. (Kyrtsou and Terraza, 2003)



However, in social systems: i) we do not know the exact "natural-laws-of-physics" describing the observed system, or more likely these do not exist, ii) we do not see simple periodical dynamics observed by time series (panel or big-data) of observations collected or measured and iii) the set and parameterization of the system differential equations (or partial PDEs) change continuously in time and space via a parametrically time-varying, adaptable or adjustable modeling and learning patterns of self-fulfilling herding agent behaviors (crises and bubbles) with modifying initial conditions. Consequently, we cannot propose static or dynamic statistical models, and importantly it is infeasible to produce long-term predictions. Perhaps, only short-term forecasts might be reliable, yet only in case the system is identified as chaotic with low levels of exogenous "noise" impact shocks (stochasticity).

5. CATASTROPHE THEORY AND WARFARE MODELING

Catastrophe theory deals with modeling "discontinuous jumps" of the state variables of dynamical systems, to new equilibrium values in an abrupt fashion. The most important contributions can be found in Rosser (1991) and Rosser and Barkley (2007). In the present work, I show that catastrophe theory fits real and observed data in conflict models, robustly and much better compared to commonly used warfare approaches.

The main subject of catastrophe theory is the classification of sudden jumps - or "catastrophes"- in the behavior of dynamical systems. Consider a family of one-dimensional functions which are parameterized by an n-dimensional vector $V = V(x, \alpha) \forall x \in R, \forall \alpha \in R^n$ and let V be an analytic function such that it can be written as a polynomial of the form $V(x, \alpha) = x^n + a_1 x^{n-1} + \dots + a_n x^0$, with some a_i being possibly equal to zero. For a given n, the graph of this polynomial has different geometric shapes when parameters vary. In case of $n = 4$ with $a_i = 0, i = 1,2,3,4$, the graph of x^4 is quite different from that of $x^4 + a_1 x^3$. Depending on the number of vanishinga_i, one or several "extremes" may occur. Catastrophe theory concentrates on the forms of$V(x, \alpha)$ which are structurally stable. A function with some a_i being possibly equal to zero is said to be a "structurally stable function" only if the number of cusps and manifolds do not change when some of these a_i change values. For example, the expression $h = x^4$ is not structurally stable because $h = x^4 + \alpha_1 x^3$ has additional 'extrema'. It can be shown that for $n = 4$ the polynomial $x^4 + a_1 x^2 + a_2 x$ is structurally stable, whilst for a given n it forms the "universal unfolding" or "cusps" of x^n. The number of parameters necessary to "stabilize" x^n for a given n is called the codimension of the unfolding. It is proven that for a codimension ≤ 4, exactly seven different universal unfoldings exist, namely four unfoldings for the one-dimensional case and three ones in the two-dimensional case. This is the essential result of Thom's famous classification theorem (Thom, 1969), in which the universal unfoldings are labeled "elementary catastrophes".

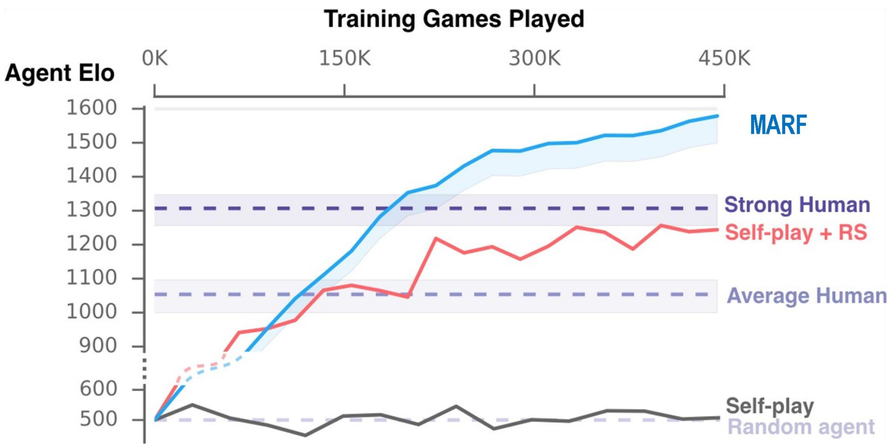
In order to demonstrate the relevance of the cusp unfoldings for the behavior of dynamical systems, I consider the system $\frac{dz}{dt} = g(z), \forall z \in R^n$, assuming that the variables can be divided into "fast" and "slow" variables. For example, let z_1 be an extremely fast variable. In that case the other variables $z_2, \ldots, z_n$ can be interpreted as "parameters" which change very slowly. The variable z_1 immediately reacts to disequilibria and moves infinitely fast to an equilibrium value, once it has been displaced from an equilibrium value. Consequently, $\frac{dz_1}{dt} = 0 = g_1(z_1, \ldots, z_n) \forall t$. The equation $g_1(z_1, \ldots, z_n) = 0$ describes an $n-1$ dimensional manifold in R^n. The idea that $\frac{das}{dt} = 0 \forall t$ implies that the motion of the system is described by the $n-1$ remaining differential equations for z_i, with $i = 2, \ldots, n$, defined to take place on the $z_1 = 0$-surface. Assume the new representation notation i.e., $z_1 = x, \alpha = (z_2, \ldots, z_n)$, and $m = n-1$. The aforementioned equation can then be written as $\frac{dx}{dt} = 0 = f(x, \alpha), \forall x \in R, \forall \alpha \in R^m$. Now, suppose that a function $F(x, \alpha)$ exists such that $F_x \equiv f(x, \alpha) = \frac{dx}{dt}$. A dynamical system which can be derived from such a function $F(x, \alpha)$ is called a gradient system.

Catastrophe theory deals with dynamical systems of the form $\frac{dx}{dt} = f(x, \alpha)$ for which $F(x, \alpha)$ is identical to a member of the family of structurally stable universal cusp-like functions $V(x, \alpha)$. We concentrate on those equilibrium surfaces that can be interpreted as the first derivative of a universal unfolding, i.e., $f(x, \alpha) = 0 = F_x(x, \alpha) \simeq V_a(x, \alpha)$. The properties of these specific equilibrium surfaces can be described by inspecting their singularity sets and their bifurcation sets. The singularity set S is defined as: $S = \{(x, \alpha) \in R \times R^m | V_{ws} = 0\}$, i.e., as the set of all (x, α) for which the second derivative of the unfolding is equal to zero. Geometrically, the singularity set consists of all parameter combinations for which the equilibrium surface is tangent to the direction of the variable x. The projection of the singularity set on the parameter space is called bifurcation set $B = \{a \in R^m | V_{wa} = 0\}$. In my work, bifurcations will be observed in the hypergame considering that I introduce a nonlinear cusp-type polynomial (equation), instead of a simple linear regression relationship used commonly thus far in Helix models.

6. DEEP REINFORCEMENT LEARNING

Reinforcement learning is a model-free Machine Learning computational technique, aiming at the automation of goal-directed decision-making in a dynamic environment (Bekiros, 2010; Kaelbling et al., 1996). The agent uses a signal to determine an optimal policy which results in maximum reward or alternatively minimum RMSE. Additionally, a maximum total utility can be attained in case Reinforcement learning is embedded into a hypergame. However, Deep Reinforcement learning topologies within games, tend to converge (learn) very slowly, as shown in a recent experiment run by DeepMind (Figure 6), using an Elo-point rating system to evaluate the resolution (Jaderberg et. al, 2019).

Figure 6. (Jaderberg et. al, 2019)



The Elo is a method of calculating the relative skill levels of players in zero-sum two-player games. A common setting starting at 500 pts represents the average human player skills. The application of Deep Reinforcement learning can also be extended by integrating agents with limited rationality (Bekiros, 2010). Relatively simple multi-agent reinforcement learning (MARL) models can often learn to cooperate effectively even in cases of complex interactions among actors. The set-up for Deep learning comprises the agent, environment, state, actions and the reward signal. In case the learning mechanism involves boundedly rational agents, the set-up utilized to "decode" the knowledge-acquisition strategy and the decision-making process of these agents is formed via rules-of-thumb for decisions under uncertainty. In a more advanced modeling approach, Bekiros (2010) proposed a novel system wherein heterogeneous agent beliefs are represented by fuzzy "Dempster–Shafer belief functions", whilst the heuristic rules are embedded as inference rules with time-varying parameters, the latter estimated by means of adaptive training.

Two main architectures of Reinforcement learning can be implemented in hypergames of conflict, namely the Deep reinforcement learning (DRL) and the Causal Generic GAI (CAI) approach, among other. An interesting set-up is attained particularly in military games, when we emulate "groupings" of AI/ML agents or substituting actors (countries), via ro(bot)-autonomous agents. Still, these DRL approaches learn once again at a very slow step when new information is incorporated into the hypergame. Potential breakthroughs can be derived from fast team MARL learning and Causal AI models that incorporate strategic adversarial training (Lee et al., 2023).

7. THE MODEL

A hybrid dynamic algorithmic hypergame approach is applied to Helix-type military warfare confrontations allowing deception and (mis)perception information strategies and deep reinforcement learning to be deployed for each actor, after a chaotic cusp-like model is used as the mechanism of conflict. Deception, belief, expectation and (mis)perception information modeling, is based upon a normative "possibilistic" approach, namely via the fuzzification of qualitative and quantitative real data samples or simulative data bootstrapped scenarios[1].

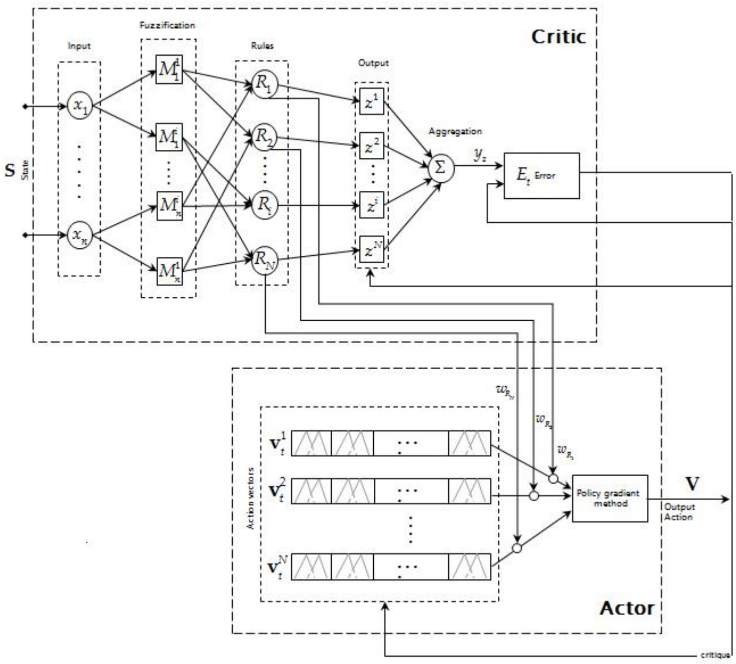
In case of a real-world paradigm incorporating big-data observation structures (matrices), the spatio-temporal multi-variate input pipeline for each actor is regularly sampled (e.g., on a daily or monthly frequency) and then is fuzzified (categorized) via a triangular or Gaussian kernel, as "inconsequential" (low impact), "neutral" or "severe". The "severe event" is located around the centroid of each actor's Dempster–Shafer belief kernel. If a data-driven model is attempted, exogenous information such as social media, news feeds, events etc., are fuzzified and transformed via sentiment analysis. At the next phase, each actor's input sample and corresponding utility function enters the cusp-like conflict mechanism, to run a fully-fledged hypergame. The resolution of the hypergame step-wise, generates utility estimations for each actor and after the utilities are de-fuzzified, the final resolution of the fully-fledged hypergame is generated.

The set-up allows for boundedly rational and irrational adversarial reasoning, and integrates the deep reinforcement learning and training approach initially introduced by Bekiros (2010).Specifically, at each learning step the agent perceives the current state of the hypergame environment via the inference rules of strategy and conflict derived by the deployment of the catastrophe polynomial, whilst the reward is represented by the prediction accuracy measured via the RMSE of their utility. Based on the state information and the reinforcement signal, each player (country) updates the "quality" of its actions and selects a response to the hypergame environment. The transition to the next state is determined by the response action, and this event-cycle continues until the end of one learning episode. The dynamics of the reinforcement learning system are determined based on three elements as described in Bekiros (2010), namely the policy (back-propagation gradient update), the value function and the reward function. In each episode, the fuzzy inference rules of information diffusion are activated in the Critic part, and the catastrophe cusp-type polynomial parameters of the model representing the adaptive expectations and preferences of each actor and of the aggregate policy function, are estimated using the Singular Value Decomposition method (SVD). During the Critic stage, the beliefs' parameters remain fixed. According to Bekiros (2010), the SVD method is applied to efficiently remove potential high "noise" levels contaminating real data, or detecting the "true" information content of the bootstrapped observations in case we run a simulative hypergame. The aforementioned are attained both in case of simulation scenarios and of real-data applications, through the principal components analysis which is incorporated in the SVD method, and

[1] The software and hardware set-up comprises the Matlab package to model and estimate cusp functions and utilities, and HAT with Java and XML for the hypergame emulation processing, using one i9 CPU. The simulation scenarios or the big-data pre-processing pipelines, the sentiment analysis and the Deep fuzzy Reinforcement Learning Actor-Critic algorithm, are coded in Python TensorFlow with CUDA scripts and data pipelines and run on two Nvidia GeForce RTX GPUs.

specifically via the calculation of the largest eigenvalues and corresponding eigenvectors[2]. Next, the output for each player and of the entire game, representing the policy of maximum utility with minimum error, is produced using the previously calculated (hyper)parameters. Lastly, the Actor part of the Deep Reinforcement learning approach, estimates the reward signal, which is sequentially used to determine the belief/perception parameter updates, based on the estimated policy of the previous episode (Bekiros, 2010).

Figure 7. (Bekiros, 2010)

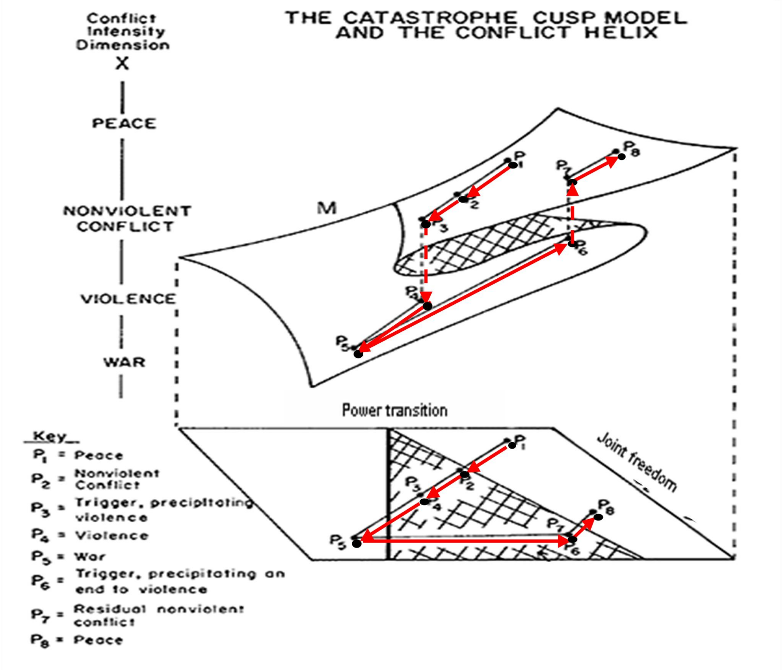


The modified Helix model comprises a novel nonlinear function to account for the conflict "intensity dimension" in order to explain the mechanism of the various temporary transitions and the evolution of the warfare mechanism. In that we could observe transitions from peace, triggering events, cusps, up to reaching conflict and back leading to resolution, so on and so forth. The hypergame runs, and primary results seem to lead consistently to an optimal utility resolution indicated by the Nash equilibrium, and which corresponds to the game-theoretical solution of "sacrifice" (volunteer's/prisoner's dilemma). The dynamic behavior in the fold catastrophe which represents the modified Helix-type conflict mechanism, encapsulates bifurcations derived by the polynomial $V(x) = x^4 + a_1x^2 + a_2x$ .Specifically, for $a_1 > 0$ no equilibrium exists, for $a_1 = 0$ a bifurcation occurs at $x = 0$, whilst for $a_1 < 0$ a stable and an unstable equilibrium branch co-exist. The simplest form applied in this work, comprises $a_1 = -1$ and $a_2 = 1$, albeit any values of $a_1 < 0$ generate almost identical jumps (see Figure 8).

[2] A hybrid hypergame based on "nowcasting" (e.g., MIDAS methodology) could also be an option, such that a model is run on real data of historical samples and observations, while it is simultaneously updated by a real-time data pipeline feed.

Figure 8.

Conflict helix model

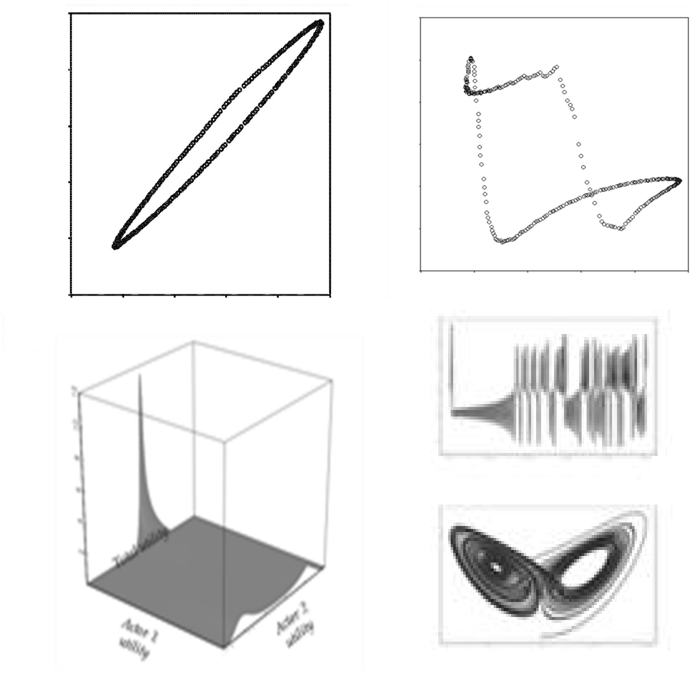


Furthermore, I conduct a stability analysis to investigate the non-equilibrium dynamics. The hypergame initiates unrestrictedly. At a next phase, I constrain the game to minimize or entirely exclude the boundedly rational last-resort adversarial conditionality, i.e., of the volunteer's dilemma, through introducing penalizing restrictions. These could be modeled via a set of binary constraints upon the linear/nonlinear hyperparameter matrix. Even highly penalizing this conditionality, once again the maximum combinatorial utility of the fully fledged hypergame is eventually provided by the same resolution. As time passes, and the next stages (episodes) of the hypergame are dynamically incorporated and estimated accordingly, the equilibrium is attained at the point of the zero-utility “sacrifice” of one actor (defeat). At the same time, periodic cycles (attractors) are observed as they’re dynamically derived from the catastrophe mechanism.

However, adding a 3rdactor in the hypergame generates a “system divergence”, in that a singular global equilibrium state no longer exists, whilst bifurcations emerge and cusps are observed on every manifold of the hyperspace[3]. Even when conditionalities are incorporated as previously, yet there is no resolution in favor of any of the two primary actors, whilst the total utility of the hypergame renders sub-optimally lower than the resolution utility of each actor when the hypergame involves only a pair of agents. The nonlinear dynamics are manifested by a “perpetual" conflict state-of-affairs bifurcating over variant manifold regions of the topological hyperspace, and through partial-equilibrium cusps and discontinuous jumps alternating with plateau periods of tranquility. Lastly, the periodic attractors disappear, and the system presents “strange” attractors with highly volatile local sub-equilibria that “explode” into different parts of the phase space (Figure 9), even in case of infinitesimal changes in parameter values and initial conditions.

[3] A real-world paradigm of a 3rd player entering the hypergame might as well be that China engages actively or the European Union (EU) intervenes upon the conflict intensity dimension directly or through sanctions, within the context of a past USA-Soviet Union state of affairs, or alongside the Ukraine-Russia conflict.

Figure 9.



8. CONCLUSIONS

In a multi-polar world, when three or more actors enter a conflict hypergame system, oscillations could evolve in an uncontrollable fashion, yet not due to exogenous shocks but mainly due to endogenous perturbations of inherent nonlinear nature. Disequilibrium could entail partial and "fragile" states (sub-equilibria) of a temporal intensity dimension, which are also unforeseeable especially when added "stochastic" exogenous factors hit (contaminate) the system one-off or pertain persistently. A total or partial equilibrium remains the least probable outcome, in a power hypergame involving multiple actors. Instead, the most probable resolution scenario entails exploding cusp-type partial equilibria observed all over the phase space with inherently unpredictable oscillations. The endogenous perturbations become even more "violent", discontinuous and unforeseeable in the presence of uncertainty. Stochastic shocks could be triggered anytime and anywhere in the world by variant exogenous factors which emerge out of the highly unstable international environment, particularly at present times. However, this outcome can be utilized to infer on the temporal forecastability of the conflict hypergame system. Taking into account that the multi-actor system entails chaotic features, based on Landau (1944), Hopf (1948), Takens (1981) and Rosser (1991), it is relatively feasible to attain short-term reliable predictions. Yet, long-term predictability is extremely limited and erratic due to high sensitivity of initial conditions, phase-space perturbations and discontinuously oscillating disequilibrium states. In his last book, Kissinger (2022) stated that: *"During Nixon years, the U.S. managed to pertain equilibrium by triangulating the two adversaries. But engaging with more, you can't just split them off and turn them against each other ... all you can do is not to accelerate the tensions, and to create options, and for that you have to have some purpose. Otherwise, this might be the tipping perturbation before an avalanche effect."* Ergo, the identical "edge-of-chaos" criticality effect which unfolds right out of the portrayed model.

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