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Original Study

Some fractional calculus findings associated with the product of incomplete ℵ-function and Srivastava polynomials

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Abstract

The generalized fractional calculus operators introduced by Saigo and Maeda in 1996 will be examined and further explored in this paper. By combining an incomplete ℵ-function with a broad category of polynomials, we create generalized fractional calculus formulations. The findings are presented in a concise manner that are helpful in creating certain lists of fractional calculus operators. The derived outcomes of a generic nature may yield results in the form of various special functions and in the form of different polynomials as special instances of the primary findings.

Keywords: Incomplete gamma function, incomplete ℵ-functions, fractional operator, Srivastava polynomial.
AMS 2020 codes: 33B20; 33C60; 26A33; 33C45.

1 Introduction

Fractional calculus (FC) refers to the study of differential and integral operators of either real or complex order. Due to a variety of applications across several scientific disciplines and technology, FC has grown in significance as well as usage over the past four decades. Fractional operators were conceived and mathematically formalized only in recent years. The numerous properties of fractional operators have generated a great deal of interest in fractional calculus in recent years, as well as a wide range of applications, with a focus on the simulation of physical issues. Areas that have seen the largest number of applications include the formulation of constitutive equations for viscoelastic materials [1], transport processes in complex media [2], mechanics [3], non-local elasticity, plasticity [4], model-order reduction of lumped parameter systems and biomedical

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engineering [5,6].

Numerous subfields of computational mathematics have found major significance in the fractional integral operator (FIO) [7], which involves a variety of special functions. Over the last five decades, several scientists like Saxena and Srivastava [8], Bhatta and Debnath [9], Saigo [10], Marichev and Kilbas [11], Ross and Miller [12], Purohit and Jangid [13], Love [14] and Ram and Kumar [15] have thoroughly investigated the characteristics, uses, and numerous extensions of several hypergeometric operators of fractional integration. Engineers, physicists, biologists and financial analysts are only some of the communities that may find several points of interest and material for further considerations in this work.

A significant number of new and recognised outcomes including Saigo FC operators and many special functions, particularly the incomplete H-function and incomplete I-function, follow as special instances of the primary discoveries. This is due to the broad scope of the Merichev-Saigo-Maeda (MSM) operators, incomplete $\aleph$-function, and a broad category of polynomials.

The rest of this paper is organized as follows. In section 2, the preliminaries are presented. In section 3, incomplete $\aleph$-functions and the Srivastava polynomial are combined, and MSM fractional order integrals of the left- and right-hand types are created. In section 4, incomplete $\aleph$-functions and the Srivastava polynomial are combined, and MSM fractional order derivative of the left- and right-hand types are created. In section 5, we develop the particular instances for the incomplete $\aleph$-functions. In section 6, the paper is completed by presenting the main contribution of the paper.

2 Preliminaries

The well-known lower and upper gamma functions of incomplete type [16] $\gamma(\mathfrak{v},\mathfrak{Y})$ and $\Gamma(\mathfrak{v},\mathfrak{Y})$ respectively, are presented as:

$$\gamma(\mathfrak{v},\mathfrak{Y}) = \int_0^{\mathfrak{Y}} \mathfrak{u}^{\mathfrak{v}-1}\, e^{-\mathfrak{u}}\, d\mathfrak{u}, \qquad \big(\Re(\mathfrak{v})>0;\ \mathfrak{Y}\geqq 0\big), \tag{1}$$

and

$$\Gamma(\mathfrak{v},\mathfrak{Y}) = \int_{\mathfrak{Y}}^{\infty} \mathfrak{u}^{\mathfrak{v}-1}\, e^{-\mathfrak{u}}\, d\mathfrak{u}, \qquad \big(\mathfrak{Y}\geqq 0;\ \Re(\mathfrak{v})>0 \quad \text{when} \quad \mathfrak{Y}=0\big). \tag{2}$$

The following connection (sometimes referred to as the decomposition formula) is satisfied by these incomplete gamma functions.

$$\gamma(\mathfrak{v},\mathfrak{Y}) + \Gamma(\mathfrak{v},\mathfrak{Y}) = \Gamma(\mathfrak{v}), \qquad \big(\Re(\mathfrak{v})>0\big). \tag{3}$$

The Srivastava investigated a broad category of polynomials [17], which is described as follows (see [18] also):

$$S_{\mathfrak{Q}}^{\mathfrak{P}}[t] = \sum_{\mathfrak{O}=0}^{[\mathfrak{Q}/\mathfrak{P}]} \frac{(-\mathfrak{Q})_{\mathfrak{P}\mathfrak{O}}}{\mathfrak{O}!} A_{\mathfrak{Q},\mathfrak{O}}\, t^{\mathfrak{O}}, \tag{4}$$

where $\mathfrak{P}\in\mathbb{Z}^+$ and $A_{\mathfrak{Q},\mathfrak{O}}$ are real or complex numbers arbitrary constants.

The notations $[k]$ indicates the floor function and $(\kappa)_\mu$ denote the Pochhammer symbol described by:

$$(\kappa)_0 = 1 \qquad \textit{and} \qquad (\kappa)_\mu = \frac{\Gamma(\kappa+\mu)}{\Gamma(\kappa)}, \qquad (\mu\in\mathbb{C}),$$

in the form of the Gamma function. Numerous FC results relating to the incomplete ℵ-functions are presented in this paper. For $\varsigma, \varsigma', \varkappa, \varkappa', \varpi \in \mathbb{C}$ and $x > 0$ with $\Re(\varpi) > 0$, the MSM FIO [19] with the left-and right-hand sides are explained as:

$$\left(\mathscr{I}_{0+}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi} f\right)(x) = \frac{x^{-\varsigma}}{\Gamma(\varpi)} \int_0^x (x-y)^{\varpi-1} y^{-\varsigma'} \times F_3\left(\varsigma, \varsigma', \varkappa, \varkappa'; \varpi; 1-\frac{y}{x}, 1-\frac{x}{y}\right) f(y)dy, \tag{5}$$

and

$$\left(\mathscr{I}_{-}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi} f\right)(x) = \frac{x^{-\varsigma'}}{\Gamma(\varpi)} \int_x^\infty (y-x)^{\varpi-1} y^{-\varsigma} \times F_3\left(\varsigma, \varsigma', \varkappa, \varkappa'; \varpi; 1-\frac{x}{y}, 1-\frac{y}{x}\right) f(y)dy, \tag{6}$$

respectively.

According to a description, the left-and right-hand handed MSM fractional differential operators are (see [20]):

$$\left(\mathscr{D}_{0+}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi} f\right)(x) = \left(\frac{d}{dx}\right)^{\alpha} \left(\mathscr{I}_{0+}^{-\varsigma',-\varsigma,-\varkappa'+\alpha,-\varkappa,-\varpi+\alpha} f\right)(x), \tag{7}$$

and

$$\left(\mathscr{D}_{-}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi} f\right)(x) = \left(-\frac{d}{dx}\right)^{[\alpha]} \left(\mathscr{I}_{-}^{-\varsigma',-\varsigma,-\varkappa',-\varkappa+\alpha,-\varpi+\alpha} f\right)(x), \tag{8}$$

where, $\alpha = [\Re(\varpi)] + 1$ and $[\Re(\varpi)]$ represent the integer component in $\Re(\varpi)$. For max $\{|x|, |y|\} < 1$, the third Appell function F_3 has the following definition:

$$F_3(\varsigma, \varsigma', \varkappa, \varkappa'; \varpi; x; y) = \sum_{i,j=0}^{\infty} \frac{(\varsigma)_i (\varsigma')_j (\varkappa)_i (\varkappa')_j}{(\varpi)_{i+j}} \frac{x^i y^j}{i!\, j!}, \tag{9}$$

here, $(\varsigma)_n$ is the Pochhammer symbol. Current articles [21, 22] include a comprehensive demonstration associated with the MSM operators along with the uses and characteristics. Saigo [10] instigate the fractional operators related with the Gauss hypergeometric function ${}_2F_1(\,)$. The left-and right-handed Saigo FIO are given the following descriptions for $\varsigma, \varkappa, \varpi \in \mathbb{C}$, $x > 0$ and $\Re(\varsigma) > 0$.

$$\left(\mathscr{I}_{0+}^{\varsigma,\varkappa,\varpi} f\right)(x) = \frac{x^{-\varsigma-\varkappa}}{\Gamma(\varsigma)} \int_0^x (x-y)^{\varsigma-1} {}_2F_1\left(\varsigma+\varkappa, -\varpi; \varsigma; 1-\frac{y}{x}\right) f(y)dy, \tag{10}$$

and

$$\left(\mathscr{I}_{-}^{\varsigma,\varkappa,\varpi} f\right)(x) = \frac{1}{\Gamma(\varsigma)} \int_x^\infty (y-x)^{\varsigma-1} y^{-\varsigma-\varkappa} {}_2F_1\left(\varsigma+\varkappa, -\varpi; \varsigma; 1-\frac{x}{y}\right) f(y)dy, \tag{11}$$

respectively.

The following definitions are given for the left-and right-sided Saigo differential operators:

$$\left(\mathscr{D}_{0+}^{\varsigma,\varkappa,\varpi} f\right)(x) = \left(\frac{d}{dx}\right)^{[\Re(\varsigma)]+1} \left(\mathscr{I}_{0+}^{-\varsigma+[\Re(\varsigma)]+1,-\varkappa-[\Re(\varsigma)]-1,\varsigma+\varpi-[\Re(\varsigma)]-1} f\right)(x), \tag{12}$$

and

$$\left(\mathscr{D}_{-}^{\varsigma,\varkappa,\varpi} f\right)(x) = \left(-\frac{d}{dx}\right)^{[\Re(\varsigma)]+1} \left(\mathscr{I}_{-}^{-\varsigma+[\Re(\varsigma)]+1,-\varkappa-[\Re(\varsigma)]-1,\varsigma+\varpi} f\right)(x). \tag{13}$$

For $\varkappa = -\varsigma$ and $\varkappa = 0$ in (10)-(13), the Riemann-Liouville (R-L) and Erdélyi-Kober (E-K) fractional operators are attained respectively (for further explanation see [23]). ${}_2F_1$ is associated with F_3 as

$$F_3(\varsigma, \gamma-\varsigma, \varkappa, \gamma-\varkappa; \gamma; x; y) = {}_2F_1(\varsigma, \varkappa; \gamma; x+y-xy).$$

The MSM fractional operators (5)-(8) are associated to Saigo operators (10)-(13) by

$$\left(\mathscr{I}_{0+}^{\varsigma,0,\varkappa,\varkappa',\varpi} f\right)(x) = \left(\mathscr{I}_{0+}^{\varpi,\varsigma-\varpi,-\varkappa} f\right)(x), \tag{14}$$

$$\left(\mathscr{I}_{-}^{\varsigma,0,\varkappa,\varkappa',\varpi} f\right)(x) = \left(\mathscr{I}_{-}^{\varpi,\varsigma-\varpi,-\varkappa} f\right)(x), \tag{15}$$

and

$$\left(\mathscr{D}_{0+}^{0,\varsigma',\varkappa,\varkappa',\varpi} f\right)(x) = \left(\mathscr{D}_{0+}^{\varpi,\varsigma'-\varpi,\varkappa'-\varpi} f\right)(x), \tag{16}$$

$$\left(\mathscr{D}_{-}^{0,\varsigma',\varkappa,\varkappa',\varpi} f\right)(x) = \left(\mathscr{D}_{-}^{\varpi,\varsigma'-\varpi,\varkappa'-\varpi} f\right)(x). \tag{17}$$

Lemma 1. *Let* $\varsigma, \varsigma', \varkappa, \varkappa', \varpi, \lambda \in \mathbb{C}$ *and* $\Re(\varpi) > 0$.

(a) If $\Re(\lambda) > max\,\{0, \Re(\varsigma'-\varkappa'), \Re(\varsigma+\varsigma'+\varkappa-\varpi)\}$, *then*

$$\left(\mathscr{I}_{0+}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi} t^{\lambda-1}\right)(x) = x^{-\varsigma-\varsigma'+\varpi+\lambda-1} \frac{\Gamma(\lambda)\Gamma(-\varsigma'+\varkappa'+\lambda)\Gamma(-\varsigma-\varsigma'-\varkappa+\varpi+\lambda)}{\Gamma(\varkappa'+\lambda)\Gamma(-\varsigma-\varsigma'+\varpi+\lambda)\Gamma(-\varsigma'-\varkappa+\varpi+\lambda)}. \tag{18}$$

(b) If $\Re(\lambda) > max\,\{\Re(\varkappa), \Re(-\varsigma-\varsigma'+\varpi), \Re(-\varsigma-\varkappa'+\varpi)\}$, *then*

$$\left(\mathscr{I}_{-}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi} t^{-\lambda}\right)(x) = x^{-\varsigma-\varsigma'+\varpi-\lambda} \frac{\Gamma(-\varkappa+\lambda)\Gamma(\varsigma+\varsigma'-\varpi+\lambda)\Gamma(\varsigma+\varkappa'-\varpi+\lambda)}{\Gamma(\lambda)\Gamma(\varsigma-\varkappa+\lambda)\Gamma(\varsigma+\varsigma'+\varkappa'-\varpi+\lambda)}. \tag{19}$$

Lemma 2. *Let* $\varsigma, \varsigma', \varkappa, \varkappa', \varpi, \lambda \in \mathbb{C}$.

(a) If $\Re(\lambda) > max\,\{0, \Re(-\varsigma+\varkappa), \Re(-\varsigma-\varsigma'-\varkappa'+\varpi)\}$, *then*

$$\left(\mathscr{D}_{0+}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi} t^{\lambda-1}\right)(x) = x^{\varsigma+\varsigma'-\varpi+\lambda-1} \frac{\Gamma(\lambda)\Gamma(\varsigma-\varkappa+\lambda)\Gamma(\varsigma+\varsigma'+\varkappa'-\varpi+\lambda)}{\Gamma(-\varkappa+\lambda)\Gamma(\varsigma+\varsigma'-\varpi+\lambda)\Gamma(\varsigma+\varkappa'-\varpi+\lambda)}. \tag{20}$$

(b) If $\Re(\lambda) > max\,\{\Re(-\varkappa'), \Re(\varsigma'+\varkappa-\varpi), \Re(\varsigma+\varsigma'-\varpi)+[\Re(\varpi)]+1\}$, *then*

$$\left(\mathscr{D}_{-}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi} t^{-\lambda}\right)(x) = x^{\varsigma+\varsigma'-\varpi-\lambda} \frac{\Gamma(\varkappa'+\lambda)\Gamma(-\varsigma-\varsigma'+\varpi+\lambda)\Gamma(-\varsigma'-\varkappa+\varpi+\lambda)}{\Gamma(\lambda)\Gamma(-\varsigma'+\varkappa'+\lambda)\Gamma(-\varsigma-\varsigma'-\varkappa+\varpi+\lambda)}. \tag{21}$$

2.1 Incomplete ℵ-function

In this paper, we introduced the incomplete ℵ- function ${}^{\Gamma}\aleph^{U,V}_{r_j,s_j,\rho_j;m}(\mathscr{Z})$ and ${}^{\gamma}\aleph^{U,V}_{r_j,s_j,\rho_j;m}(\mathscr{Z})$ [24, 25] as follows:

$$\begin{aligned}{}^{\gamma}\aleph^{U,V}_{r_j,s_j,\rho_j;m}(\mathscr{Z}) &= {}^{\gamma}\aleph^{U,V}_{r_j,s_j,\rho_j;m}\left[\mathscr{Z}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]\\&= \frac{1}{2\pi\iota}\int_{\$}\Phi(q,\mathscr{Y})\,\mathscr{Z}^{-q}\,dq,\end{aligned} \tag{22}$$

where

$$\Phi(q,\mathscr{Y})=\frac{\gamma(1-\Lambda_1-\mathfrak{D}_1q;\mathscr{Y})\prod_{n=1}^{U}\Gamma(\varepsilon_n+\mathfrak{E}_nq)\prod_{n=2}^{V}\Gamma(1-\Lambda_n-\mathfrak{D}_nq)}{\sum_{j=1}^{m}\rho_j\left[\prod_{n=U+1}^{s_j}\Gamma(1-\varepsilon_{nj}-\mathfrak{E}_{nj}q)\prod_{n=V+1}^{r_j}\Gamma(\Lambda_{nj}+\mathfrak{D}_{nj})\right]},\tag{23}$$

and

$$\begin{aligned}{}^{\Gamma}\aleph^{U,V}_{r_j,s_j,\rho_j;m}(\mathscr{Z})&={}^{\Gamma}\aleph^{U,V}_{r_j,s_j,\rho_j;m}\left[\mathscr{Z}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]\\&=\frac{1}{2\pi\imath}\int_{\$}\Psi(q,\mathscr{Y})\,\mathscr{Z}^{-q}\,dq,\end{aligned}\tag{24}$$

where

$$\Psi(q,\mathscr{Y})=\frac{\Gamma(1-\Lambda_1-\mathfrak{D}_1q;\mathscr{Y})\prod_{n=1}^{U}\Gamma(\varepsilon_n+\mathfrak{E}_nq)\prod_{n=2}^{V}\Gamma(1-\Lambda_n-\mathfrak{D}_nq)}{\sum_{j=1}^{m}\rho_j\left[\prod_{n=U+1}^{s_j}\Gamma(1-\varepsilon_{nj}-\mathfrak{E}_{nj}q)\prod_{n=V+1}^{r_j}\Gamma(\Lambda_{nj}+\mathfrak{D}_{nj})\right]},\tag{25}$$

for $\mathscr{Z}\neq 0$, $\mathscr{Y}\geqq 0$, the incomplete ℵ-functions ${}^{\gamma}\aleph^{U,V}_{r_j,s_j,\rho_j;m}(\mathscr{Z})$ and ${}^{\Gamma}\aleph^{U,V}_{r_j,s_j,\rho_j;m}(\mathscr{Z})$ in (22) and (24) exist in the circumstances listed as follows:

The complex- plane contour \$ extended from $\gamma-i\infty$ to $\gamma+i\infty$, $\gamma\in\mathbb{R}$, and the poles of the gamma functions $\Gamma(1-\Lambda_n-\mathfrak{D}_nq)$ for $n=1,2,\cdots,V$ are not perfectly matched with the gamma function poles $\Gamma(\varepsilon_n+\mathfrak{E}_nq)$ for $n=1,2,..,U$. The parameters r_j and $s_j\in\mathbb{Z}^+$ satisfying $0\leq V\leq r_j, 0\leq U\leq s_j$ for $1\leq j\leq m$. The parameters $\mathfrak{D}_n$, $\mathfrak{E}_n$, $\mathfrak{E}_{nj}$, $\mathfrak{D}_{nj}$ are positive numbers, and Λ_n, ε_n, Λ_{nj}, ε_{nj} are complex. The void product is considered to represent unity and all of the poles $\Phi(q,\mathscr{Y})$ and $\Psi(q,\mathscr{Y})$ should be simple.

A number of unique remarks are made about incomplete ℵ-functions and are as follows:

Remark 1. When $\mathscr{Y}=0$, Equation (24) changes to the suggested ℵ-function of Sudland [26, 27]:

$$\begin{aligned}&{}^{\Gamma}\aleph^{U,V}_{r_j,s_j,\rho_j;m}\left[\mathscr{Z}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:0),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]\\&=\aleph^{U,V}_{r_j,s_j,\rho_j;m}\left[\mathscr{Z}\left|\begin{matrix}(\Lambda_n,\mathfrak{D}_n)_{1,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right].\end{aligned}\tag{26}$$

Remark 2. Again, when $\rho_j=1$ in (22) and (24), then it changes to the incomplete I-function of Bansal and Kumar [28]:

$$\begin{aligned}&{}^{\gamma}\aleph^{U,V}_{r_j,s_j,\rho_j;m}\left[\mathscr{Z}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[1(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[1(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]\\&={}^{\gamma}I^{U,V}_{r_j,s_j;m}\left[\mathscr{Z}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},(\Lambda_{nj},\mathfrak{D}_{nj})_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},(\varepsilon_{nj},\mathfrak{E}_{nj})_{U+1,s_j}\end{matrix}\right.\right],\end{aligned}\tag{27}$$

and

$$
{}^{\Gamma}\aleph^{U,V}_{r_j,s_j,\rho_j;m}\left[\mathscr{Z}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[1(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[1(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]
= {}^{\Gamma}I^{U,V}_{r_j,s_j;m}\left[\mathscr{Z}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},(\Lambda_{nj},\mathfrak{D}_{nj})_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},(\varepsilon_{nj},\mathfrak{E}_{nj})_{U+1,s_j}\end{matrix}\right.\right]. \tag{28}
$$

Remark 3. Next, taking $\mathscr{Y}=0$ and $\rho_j=1$ in (24), then it turns into the Saxena I-function [29]:

$$
{}^{\Gamma}\aleph^{U,V}_{r_j,s_j,1;m}\left[\mathscr{Z}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:0),(\Lambda_n,\mathfrak{D}_n)_{2,V},[1(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[1(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]
= I^{U,V}_{r_j,s_j;m}\left[\mathscr{Z}\left|\begin{matrix}(\Lambda_n,\mathfrak{D}_n)_{1,V},(\Lambda_{nj},\mathfrak{D}_{nj})_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},(\varepsilon_{nj},\mathfrak{E}_{nj})_{U+1,s_j}\end{matrix}\right.\right]. \tag{29}
$$

Remark 4. Further taking $\rho_j=1$ and $m=1$ in (22) and (24), then it turns into the incomplete H-function(see [30, 31] also) of Srivastava [32]:

$$
{}^{\gamma}\aleph^{U,V}_{r_j,s_j,1;1}\left[\mathscr{Z}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[1(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[1(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]
= \gamma^{U,V}_{r,s}\left[\mathscr{Z}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,r}\\(\varepsilon_n,\mathfrak{E}_n)_{1,s}\end{matrix}\right.\right], \tag{30}
$$

and

$$
{}^{\Gamma}\aleph^{U,V}_{r_j,s_j,1;1}\left[\mathscr{Z}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[1(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[1(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]
= \Gamma^{U,V}_{r,s}\left[\mathscr{Z}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,r}\\(\varepsilon_n,\mathfrak{E}_n)_{1,s}\end{matrix}\right.\right]. \tag{31}
$$

Remark 5. Next, we take $\mathscr{Y}=0$, $\rho_j=1$, and $m=1$ in (24), then it turns into the H-function of Srivastava [33]:

$$
{}^{\Gamma}\aleph^{U,V}_{r_j,s_j,1;1}\left[\mathscr{Z}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:0),(\Lambda_n,\mathfrak{D}_n)_{2,V},[1(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[1(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]
= H^{U,V}_{r,s}\left[\mathscr{Z}\left|\begin{matrix}(\Lambda_n,\mathfrak{D}_n)_{1,r}\\(\varepsilon_n,\mathfrak{E}_n)_{1,s}\end{matrix}\right.\right]. \tag{32}
$$

We developed the FC findings linked to the incomplete $\aleph$-functions, which were influenced by the work of Srivastava et al. [34].

3 Fractional integral formulas

In this part, we create two formulas for fractional integrals that multiply incomplete $\aleph$-functions and the generic class of polynomials specified in equation (24) and (4), respectively.

Theorem 3. *Let* $\varsigma,\ \varsigma',\ \varkappa,\ \varkappa',\ \varpi, z, \alpha \in \mathbb{C}$ *and* $\Re(\varpi) > 0, \mu > 0, \lambda_k > 0\ (k = 1,2,3,\cdots,s)$,

$$\Re(\alpha) + \mu \min_{1\le j\le U} \Re\left(\frac{\varepsilon_j}{\varkappa_j}\right) > max[0, \Re(\varsigma+\varsigma'+\varkappa-\varpi), \Re(\varsigma'-\varkappa')].$$

Further the parameters $r_j,\ s_j, \mathfrak{P} \in \mathbb{Z}^+$ *satisfying* $0 \le V \le r_j, 0 \le U \le s_j$ *for* $j = 1,2,\cdots m$, $\mathfrak{D}_n, \mathfrak{E}_n, \mathfrak{E}_{nj}, \mathfrak{D}_{nj} \in \mathbb{R}^+$, $\Lambda_n, \varepsilon_n, \Lambda_{nj}, \varepsilon_{nj} \in \mathbb{C}\ (j = 1,2,\cdots,r_j; n = 1,2,\cdots,s_j)$, $A^k_{\mathfrak{Q}_k,\mathfrak{P}_k}$ *are real or complex numbers arbitrary constant for* $k = 1,2,\cdots,s$ *and* $\rho_i > 0$ *for* $i = 1,2,\cdots,m$, *then the outcome shown below is accurate:*

$$\mathscr{I}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi}_{0+}\left(t^{\alpha-1}\prod_{j=1}^{s} S^{\mathfrak{P}_j}_{\mathfrak{Q}_j}[c_j t^{\lambda_j}]\right.$$
$$\left.\times\, {}^{\Gamma}\aleph^{U,V}_{r_j,s_j,\rho_j;m}\left[zt^{\mu}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]\right)(x)$$
$$= x^{\alpha-\varsigma-\varsigma'+\varpi-1}\sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}A^{(1)}_{\mathfrak{Q}_1,\mathfrak{P}_1}\cdots A^{(s)}_{\mathfrak{Q}_s,\mathfrak{P}_s}\,c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}\,(x)^{\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j}$$
$$\times\, {}^{\Gamma}\aleph^{U,V+3}_{r_j+3,s_j+3,\rho_j;m}\left[zx^{\mu}\left|\begin{matrix}(1-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1-\alpha+\varsigma+\varsigma'+\varkappa-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},(1-\alpha+\varsigma+\varsigma'-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\end{matrix}\right.\right.$$
$$\left.\begin{matrix}(1-\alpha+\varsigma'-\varkappa'-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(1-\alpha-\varkappa'-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1+\varsigma'+\varkappa-\varpi-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right]. \tag{33}$$

Proof. The LHS of equation (33) is:

$$T_1 = \mathscr{I}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi}_{0+}\left(t^{\alpha-1}\prod_{j=1}^{s} S^{\mathfrak{P}_j}_{\mathfrak{Q}_j}[c_j t^{\lambda_j}]\right.$$
$$\left.\times\, {}^{\Gamma}\aleph^{U,V}_{r_j,s_j,\rho_j;m}\left[zt^{\mu}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]\right). \tag{34}$$

Replace the incomplete ℵ-function and Srivastava polynomial by equation (24) and (4) respectively and by reversing the summation order, we discover the subsequent form:

$$T_1 = \sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}\times A^{(1)}_{\mathfrak{Q}_1,\mathfrak{P}_1}\cdots A^{(s)}_{\mathfrak{Q}_s,\mathfrak{P}_s}\,c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}$$
$$\times\frac{1}{2\pi\imath}\int_{\$}\Psi(q,\mathscr{Y})\,z^{-q}\left(\mathscr{I}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi}_{0+}\,t^{\alpha+\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j-\mu q-1}\right)(x)dq, \tag{35}$$

where $\Psi(q,\mathscr{Y})$ is defined in equation (25).

Using equation (18) of Lemma 1, we discover the subsequent form:

$$T_1 = \sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}\times A^{(1)}_{\mathfrak{Q}_1,\mathfrak{P}_1}\cdots A^{(s)}_{\mathfrak{Q}_s,\mathfrak{P}_s}\,c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}$$
$$\times\frac{1}{2\pi\imath}\int_{\$}x^{\alpha-\varsigma-\varsigma'+\varpi+\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j-1}\,\Psi(q,\mathscr{Y})\,(zx^{\mu})^{-q}\frac{\Gamma(\alpha+\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j-\mu q)}{\Gamma(\varkappa'+\alpha+\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j-\mu q)}\times$$
$$\frac{\Gamma(-\varsigma'+\varkappa'+\alpha+\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j-\mu q)\,\Gamma(-\varsigma-\varsigma'-\varkappa+\varpi+\alpha+\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j-\mu q)}{\Gamma(-\varsigma-\varsigma'+\varpi+\alpha+\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j-\mu q)\,\Gamma(-\varsigma'-\varkappa+\varpi+\alpha+\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j-\mu q)}dq. \tag{36}$$

Finally, after some adjustment of terms , we obtain RHS of equation (33).

Theorem 4. *Let* $\varsigma, \varsigma', \varkappa, \varkappa', \varpi, z, \alpha \in \mathbb{C}$ *and* $\Re(\varpi) > 0, \mu > 0, \lambda_k > 0$ $(k = 1,2,3,\cdots,s)$,

$$\Re(\alpha) + \mu \min_{1\le j\le U} \Re\left(\frac{\varepsilon_j}{\varkappa_j}\right) > max[0, \Re(\varsigma + \varsigma' + \varkappa - \varpi), \Re(\varsigma' - \varkappa')].$$

Further the parameters r_j, $s_j, \mathfrak{P} \in \mathbb{Z}^+$ *satisfying* $0 \le V \le r_j, 0 \le U \le s_j$ *for* $j = 1,2,\cdots m$, $\mathfrak{D}_n, \mathfrak{E}_n, \mathfrak{E}_{nj}, \mathfrak{D}_{nj} \in \mathbb{R}^+$, $\Lambda_n, \varepsilon_n, \Lambda_{nj}, \varepsilon_{nj} \in \mathbb{C}$ $(j = 1,2,\cdots,r_j; n = 1,2,\cdots,s_j)$, $A^k_{\mathfrak{Q}_k,\mathfrak{P}_k}$ *are real or complex numbers arbitrary constant for* $k = 1,2,\cdots,s$ *and* $\rho_i > 0$ *for* $i = 1,2,\cdots,m$, *then the outcome shown below is accurate:*

$$\begin{aligned}
&\mathscr{I}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi}_{0+}\Bigg(t^{\alpha-1}\prod_{j=1}^{s} S^{\mathfrak{P}_j}_{\mathfrak{Q}_j}[c_j t^{\lambda_j}] \\
&{}^{\gamma}\aleph^{U,V}_{r_j,s_j,\rho_j;m}\left[zt^{\mu}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]\Bigg)(x)\\
&= x^{\alpha-\varsigma-\varsigma'+\varpi-1}\sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}A^{(1)}_{\mathfrak{Q}_1,\mathfrak{P}_1}\cdots A^{(s)}_{\mathfrak{Q}_s,\mathfrak{P}_s}\,c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}\,(x)^{\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j}\\
&\times{}^{\gamma}\aleph^{U,V+3}_{r_j+3,s_j+3,\rho_j;m}\left[zx^{\mu}\left|\begin{matrix}(1-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1-\alpha+\varsigma+\varsigma'+\varkappa-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},(1-\alpha+\varsigma+\varsigma'-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\end{matrix}\right.\right.\\
&\left.\begin{matrix}(1-\alpha+\varsigma'-\varkappa'-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(1-\alpha-\varkappa'-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1+\varsigma'+\varkappa-\varpi-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right]. \qquad (37)
\end{aligned}$$

Proof. Theorem 4 is proved in the same manner as Theorem 3 with the same conditions.

The following corollary is obtained regarding the Saigo FIO [10] in light of the equation (14).

Corollary 5. *Let* $\varsigma, \varkappa, \varpi, z, \alpha \in \mathbb{C}$ *and* $\Re(\varsigma) > 0, \mu > 0, \lambda_k > 0$ $(k = 1,2,\cdots,s)$,

$$\Re(\alpha) + \mu \min_{1\le j\le U} \Re\left(\frac{\varepsilon_j}{\varkappa_j}\right) > max[0, \Re(\varkappa - \varpi)].$$

Further the parameters r_j, $s_j, \mathfrak{P} \in \mathbb{Z}^+$ *satisfying* $0 \le V \le r_j$, $0 \le U \le s_j$ *for* $j = 1,2,\cdots m$, $\mathfrak{D}_n, \mathfrak{E}_n, \mathfrak{E}_{nj}, \mathfrak{D}_{nj} \in \mathbb{R}^+$, $\Lambda_n, \varepsilon_n, \Lambda_{nj}, \varepsilon_{nj} \in \mathbb{C}$ $(j = 1,2,,\cdots,r_j; n = 1,2,\cdots,s_j)$, $A^k_{\mathfrak{Q}_k,\mathfrak{P}_k}$ *are real or complex numbers arbitrary constant for* $k = 1,2,3,\cdots,s$ *and* $\rho_i > 0$ *for* $i = 1,2,\cdots,m$, *then the outcome shown below is accurate:*

$$\begin{aligned}
&\mathscr{I}^{\varsigma,\varkappa,\varpi}_{0+}\Bigg(t^{\alpha-1}\prod_{j=1}^{s} S^{\mathfrak{P}_j}_{\mathfrak{Q}_j}[c_j t^{\lambda_j}] \\
&{}^{\Gamma}\aleph^{U,V}_{r_j,s_j,\rho_j;m}\left[zt^{\mu}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]\Bigg)(x)\\
&= x^{\alpha-\varkappa-1}\sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}A^{(1)}_{\mathfrak{Q}_1,\mathfrak{P}_1}\cdots A^{(s)}_{\mathfrak{Q}_s,\mathfrak{P}_s}\,c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}\,(x)^{\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j}\\
&\times{}^{\Gamma}\aleph^{U,V+2}_{r_j+2,s_j+3,\rho_j;m}\left[zx^{\mu}\left|\begin{matrix}(1-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1-\alpha+\varkappa-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},(1-\alpha-\varsigma-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\end{matrix}\right.\right.\\
&\left.\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(1-\alpha+\varkappa-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right]. \qquad (38)
\end{aligned}$$

The same result can be obtained concerning Saigo FIO for the lower incomplete $\aleph$- function.

Remark 6. By substituting $\varkappa = -\varsigma$ and $\varkappa = 0$ in Corollary 5, respectively, we can also get findings for the fractional derivative operators of R-L and E-K.

Theorem 6. *Let $\varsigma, \varsigma', \varkappa, \varkappa', \varpi, z, \alpha \in \mathbb{C}$ and $\Re(\varpi) > 0, \mu > 0, \lambda_k > 0$ $(k = 1,2,3,\cdots,s)$,*

$$\Re(\alpha) - \mu \min_{1\le j\le U} \Re\left(\frac{\varepsilon_j}{\varkappa_j}\right) < 1 + min[\Re(-\varkappa), \Re(\varsigma+\varsigma^{'}-\varpi), \Re(\varsigma+\varkappa^{'}-\varpi)].$$

Further the parameters r_j, $s_j, \mathfrak{P} \in \mathbb{Z}^+$ satisfying $0 \le V \le r_j, 0 \le U \le s_j$ for $j = 1,2,\cdots m$, $\mathfrak{D}_n, \mathfrak{E}_n, \mathfrak{E}_{nj}, \mathfrak{D}_{nj} \in \mathbb{R}^+$, $\Lambda_n, \varepsilon_n, \Lambda_{nj}, \varepsilon_{nj} \in \mathbb{C}$ $(j = 1,2,\cdots,r_j; n = 1,2,\cdots,s_j)$, $A^k_{\mathfrak{Q}_k,\mathfrak{P}_k}$ are real or complex numbers arbitrary constant for $k = 1,2,\cdots,s$ and $\rho_i > 0$ for $i = 1,2,\cdots,m$, then the outcome shown below is accurate:

$$\begin{aligned}
&\mathscr{I}_{-}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi}\Bigg(t^{\alpha-1}\prod_{j=1}^{s} S_{\mathfrak{Q}_j}^{\mathfrak{P}_j}[c_j t^{\lambda_j}] \\
&{}^{\Gamma}\aleph^{U,V}_{r_j,s_j,\rho_j;m}\left[zt^{\mu}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\ (\varepsilon_n,\mathfrak{E}_n)_{1,U},[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]\Bigg)(x)\\
&= x^{\alpha-\varsigma-\varsigma^{'}+\varpi-1}\sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}A^{(1)}_{\mathfrak{Q}_1,\mathfrak{P}_1}\cdots A^{(s)}_{\mathfrak{Q}_s,\mathfrak{P}_s}\,c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}\,(x)^{\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j}\\
&\times{}^{\Gamma}\aleph^{U+3,V}_{r_j+3,s_j+3,\rho_j;m}\left[zx^{\mu}\left|\begin{matrix}(1-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1-\alpha+\varsigma+\varsigma^{'}+\varkappa^{'}-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\\ (\varepsilon_n,\mathfrak{E}_n)_{1,U},(1-\alpha+\varsigma+\varsigma^{'}-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\end{matrix}\right.\right.\\
&\qquad\left.\begin{matrix}(1-\alpha+\varsigma-\varkappa-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\ (1-\alpha-\varkappa-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1+\varsigma+\varkappa^{'}-\varpi-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right]. \qquad (39)
\end{aligned}$$

Proof. The LHS of equation (39) is:

$$\begin{aligned}
T_2 = {}&\mathscr{I}_{-}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi}\Bigg(t^{\alpha-1}\prod_{j=1}^{s} S_{\mathfrak{Q}_j}^{\mathfrak{P}_j}[c_j t^{\lambda_j}]\\
&{}^{\Gamma}\aleph^{U,V}_{r_j,s_j,\rho_j;m}\left[zt^{\mu}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\ (\varepsilon_n,\mathfrak{E}_n)_{1,U},[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]\Bigg). \qquad (40)
\end{aligned}$$

Replace the incomplete $\aleph$-function and Srivastava polynomial by equation (24) and (4) respectively and by reversing the summation order, we discover the subsequent form:

$$\begin{aligned}
T_2 = {}&\sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}\times A^{(1)}_{\mathfrak{Q}_1,\mathfrak{P}_1}\cdots A^{(s)}_{\mathfrak{Q}_s,\mathfrak{P}_s}\,c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}\\
&\times\frac{1}{2\pi\imath}\int_{\$}\Psi(q,\mathscr{Y})\,z^{-q}\left(\mathscr{I}_{-}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi}t^{-(-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j+\mu q+1)}\right)(x)dq, \qquad (41)
\end{aligned}$$

where $\Psi(q,\mathscr{Y})$ is defined in equation (25).

Using equation (19) of Lemma 1, we discover the subsequent form:

$$T_2 = \sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]} \cdots \sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]} \frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!} \times A^{(1)}_{\mathfrak{Q}_1,\mathfrak{P}_1}\cdots A^{(s)}_{\mathfrak{Q}_s,\mathfrak{P}_s}\, c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}$$
$$\times \frac{1}{2\pi\iota}\int_{\$} x^{\alpha-\varsigma-\varsigma'+\varpi+\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j-1}\,\Psi(q,\mathscr{Y})\,(zx^{\mu})^{-q}\frac{\Gamma(1-\alpha-\varkappa-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j+\mu q)}{\Gamma(1-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j+\mu q)}\times$$
$$\frac{\Gamma(1+\varsigma+\varsigma'-\varpi-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j+\mu q)\Gamma(1-\alpha+\varsigma+\varkappa'-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j+\mu q)}{\Gamma(1-\alpha+\varsigma-\varkappa-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j+\mu q)\Gamma(1-\alpha+\varsigma+\varsigma'+\varkappa'-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j+\mu q)}dq. \tag{42}$$

Finally, after some adjustment of terms, we obtain RHS of equation (39).

Theorem 7. *Let ς, ς', $\varkappa$, $\varkappa'$, $\varpi, z, \alpha \in \mathbb{C}$ and $\Re(\varpi) > 0, \mu > 0, \lambda_k > 0$ $(k = 1,2,3,\cdots,s)$,*

$$\Re(\alpha) - \mu \min_{1\le j\le U} \Re\left(\frac{\varepsilon_j}{\varkappa_j}\right) < 1 + min[\Re(-\varkappa), \Re(\varsigma+\varsigma'-\varpi), \Re(\varsigma+\varkappa'-\varpi)].$$

Further the parameters r_j, $s_j, \mathfrak{P} \in \mathbb{Z}^+$ satisfying $0 \le V \le r_j, 0 \le U \le s_j$ for $j = 1,2,\cdots m$, $\mathfrak{D}_n, \mathfrak{E}_n, \mathfrak{E}_{nj}, \mathfrak{D}_{nj} \in \mathbb{R}^+$, $\Lambda_n, \varepsilon_n, \Lambda_{nj}, \varepsilon_{nj} \in \mathbb{C}$ $(j = 1,2,\cdots,r_j; n = 1,2,\cdots,s_j)$, $A^k_{\mathfrak{Q}_k,\mathfrak{P}_k}$ are real or complex numbers arbitrary constant for $k = 1,2,\cdots,s$ and $\rho_i > 0$ for $i = 1,2,\cdots,m$, then the outcome shown below is accurate:

$$\mathscr{I}_{-}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi}\Bigg(t^{\alpha-1}\prod_{j=1}^{s} S^{\mathfrak{P}_j}_{\mathfrak{Q}_j}[c_j t^{\lambda_j}]$$
$$ {}^{\gamma}\aleph^{U,V}_{r_j,s_j,\rho_j;m}\left[zt^{\mu}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]\Bigg)(x)$$
$$= x^{\alpha-\varsigma-\varsigma'+\varpi-1}\sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}A^{(1)}_{\mathfrak{Q}_1,\mathfrak{P}_1}\cdots A^{(s)}_{\mathfrak{Q}_s,\mathfrak{P}_s}\,c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}\,(x)^{\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j}$$
$$\times {}^{\gamma}\aleph^{U+3,V}_{r_j+3,s_j+3,\rho_j;m}\left[zx^{\mu}\left|\begin{matrix}(1-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1-\alpha+\varsigma+\varsigma'+\varkappa'-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},(1-\alpha+\varsigma+\varsigma'-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\end{matrix}\right.\right.$$
$$\left.\begin{matrix}(1-\alpha+\varsigma-\varkappa-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(1-\alpha-\varkappa-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1+\varsigma+\varkappa'-\varpi-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right]. \tag{43}$$

Proof. Theorem 7 is proved in the same way as Theorem 6 with the same conditions. The following corollary is obtained regarding the Saigo FIO [10] in light of the equation (15).

Corollary 8. *Let ς, $\varkappa$, $\varpi, z, \alpha \in \mathbb{C}$ and $\Re(\varsigma) > 0, \mu > 0, \lambda_k > 0$ $(k = 1,2,\cdots,s)$,*

$$\Re(\alpha) - \mu \min_{1\le j\le U} \Re\left(\frac{\varepsilon_j}{\varkappa_j}\right) < 1 + min[\Re(\varkappa), \Re(\varpi)].$$

Further the parameters r_j, $s_j, \mathfrak{P} \in \mathbb{Z}^+$ satisfying $0 \le V \le r_j, 0 \le U \le s_j$ for $j = 1,2,\cdots m$, $\mathfrak{D}_n, \mathfrak{E}_n, \mathfrak{E}_{nj}, \mathfrak{D}_{nj} \in \mathbb{R}^+$, $\Lambda_n, \varepsilon_n, \Lambda_{nj}, \varepsilon_{nj} \in \mathbb{C}$ $(j = 1,2,,\cdots,r_j; n = 1,2,\cdots,s_j)$, $A^k_{\mathfrak{Q}_k,\mathfrak{P}_k}$ are real or complex numbers arbitrary constant *for $k=1,2,\cdots,s$ and $\rho_i>0$ for $i=1,2,\cdots,m$, then the outcome shown below is accurate:*

$$\mathscr{I}_{-}^{\varsigma,\varkappa,\varpi}\left(t^{\alpha-1}\prod_{j=1}^{s}S_{\mathfrak{Q}_j}^{\mathfrak{P}_j}[c_j t^{\lambda_j}]\right.$$
$${}^{\Gamma}\aleph_{r_j,s_j,\rho_j;m}^{U,V}\left[zt^{\mu}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]\Bigg)(x)$$
$$=x^{\alpha-\varkappa-1}\sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}A_{\mathfrak{Q}_1,\mathfrak{P}_1}^{(1)}\cdots A_{\mathfrak{Q}_s,\mathfrak{P}_s}^{(s)}c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}(x)^{\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j}$$
$$\times{}^{\Gamma}\aleph_{r_j+2,s_j+2,\rho_j;m}^{U+2,V}\left[zx^{\mu}\left|\begin{matrix}(1-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1-\alpha+\varsigma+\varkappa+\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},(1+\varkappa-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\end{matrix}\right.\right.$$
$$\left.\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(1-\alpha+\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right]. \tag{44}$$

The same result can be obtained concerning Saigo FIO for the lower incomplete ℵ- function.

Remark 7. By substituting $\varkappa=-\varsigma$ and $\varkappa=0$ in Corollary 8, respectively, we can also get findings for the fractional derivative operators of R-L and E-K.

4 Fractional derivative formulas

In this part, we create two formulas for fractional derivative that multiply incomplete ℵ-functions and the generic class of polynomials specified in (24) and (4), respectively.

Theorem 9. *Let $\varsigma,\varsigma',\varkappa,\varkappa',\varpi,z,\alpha\in\mathbb{C}$ and $\Re(\varpi)>0,\mu>0,\lambda_k>0$ $(k=1,2,3,\cdots,s)$,*

$$\mu\max_{1\le j\le U}\left[\frac{-\Re(\varepsilon_j)}{\mathfrak{E}_j}\right]<\Re(\alpha)+\min[0,\Re(\varsigma-\varkappa),\Re(\varsigma'+\varkappa'+\varsigma-\varpi)].$$

Further the parameters r_j, $s_j,\mathfrak{P}\in\mathbb{Z}^+$ satisfying $0\le V\le r_j, 0\le U\le s_j$ for $j=1,2,\cdots m$, $\mathfrak{D}_n,\mathfrak{E}_n,\mathfrak{E}_{nj},\mathfrak{D}_{nj}\in\mathbb{R}^+$, $\Lambda_n,\varepsilon_n,\Lambda_{nj},\varepsilon_{nj}\in\mathbb{C}$ $(j=1,2,\cdots,r_j;n=1,2,\cdots,s_j)$, $A_{\mathfrak{Q}_k,\mathfrak{P}_k}^{k}$ are real or complex numbers arbitrary constant for $k=1,2,\cdots,s$ and $\rho_i>0$ for $i=1,2,\cdots,m$, then the outcome shown below is accurate:

$$\mathscr{D}_{0+}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi}\left(t^{\alpha-1}\prod_{j=1}^{s}S_{\mathfrak{Q}_j}^{\mathfrak{P}_j}[c_j t^{\lambda_j}]\right.$$
$${}^{\Gamma}\aleph_{r_j,s_j,\rho_j;m}^{U,V}\left[zt^{\mu}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]\Bigg)(x)$$
$$=x^{\alpha+\varsigma+\varsigma'-\varpi-1}\sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}A_{\mathfrak{Q}_1,\mathfrak{P}_1}^{(1)}\cdots A_{\mathfrak{Q}_s,\mathfrak{P}_s}^{(s)}c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}(x)^{\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j}$$
$$\times{}^{\Gamma}\aleph_{r_j+3,s_j+3,\rho_j;m}^{U,V+3}\left[zx^{\mu}\left|\begin{matrix}(1-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1-\alpha-\varsigma-\varsigma'-\varkappa'+\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},(1-\varsigma-\varsigma'+\varpi-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\end{matrix}\right.\right.$$
$$\left.\begin{matrix}(1-\alpha-\varsigma+\varkappa-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(1-\alpha+\varkappa-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1-\varsigma-\varkappa'+\varpi-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right]. \tag{45}$$

Proof. The LHS of equation (45) is:

$$T_3 = \mathscr{D}_{0+}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi}\left(t^{\alpha-1}\prod_{j=1}^{s} S_{\mathfrak{Q}_j}^{\mathfrak{P}_j}[c_j t^{\lambda_j}]\right.$$
$$\left.\Gamma\aleph_{r_j,s_j,\rho_j;m}^{U,V}\left[zt^{\mu}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]\right)(x). \tag{46}$$

Replace the incomplete $\aleph$- function and Srivastava polynomial by equation (24) and (4) respectively and by reversing the summation order, we discover the subsequent form:

$$T_3 = \sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}\times A_{\mathfrak{Q}_1,\mathfrak{P}_1}^{(1)}\cdots A_{\mathfrak{Q}_s,\mathfrak{P}_s}^{(s)}c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}$$
$$\times\frac{1}{2\pi\imath}\int_{\$}\Psi(q,\mathscr{Y})\,z^{-q}\left(\mathscr{D}_{0+}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi}t^{\alpha+\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j-\mu q-1}\right)(x)dq, \tag{47}$$

where $\Psi(q,\mathscr{Y})$ is defined in equation (25).

Using equation (20) of Lemma 2, we discover the subsequent form:

$$T_3 = \sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}\times A_{\mathfrak{Q}_1,\mathfrak{P}_1}^{(1)}\cdots A_{\mathfrak{Q}_s,\mathfrak{P}_s}^{(s)}c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}$$
$$\times\frac{1}{2\pi\imath}\int_{\$}x^{\alpha-\varsigma-\varsigma'+\varpi+\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j-1}\,\Psi(q,\mathscr{Y})\,(zx^{\mu})^{-q}\frac{\Gamma(\alpha+\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j-\mu q)}{\Gamma(\alpha-\varkappa+\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j-\mu q)}\times$$
$$\frac{\Gamma(\varsigma-\varkappa+\alpha+\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j-\mu q)\Gamma(\alpha+\varsigma+\varsigma'+\varkappa'-\varpi+\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j-\mu q)}{\Gamma(\alpha+\varsigma+\varkappa'-\varpi+\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j-\mu q)\Gamma(\alpha+\varsigma+\varsigma'-\varpi+\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j-\mu q)}dq. \tag{48}$$

Finally, after some adjustment of terms, we obtain RHS of equation (45).

Theorem 10. *Let* $\varsigma,\varsigma',\varkappa,\varkappa',\varpi,z,\alpha\in\mathbb{C}$ *and* $\Re(\varpi)>0,\mu>0,\lambda_k>0\ (k=1,2,3,\cdots,s)$,

$$\mu\max_{1\le j\le U}\left[\frac{-\Re(\varepsilon_j)}{\mathfrak{E}_j}\right]<\Re(\alpha)+\min[0,\Re(\varsigma-\varkappa),\Re(\varsigma'+\varkappa'+\varsigma-\varpi)].$$

Further the parameters r_j, $s_j,\mathfrak{P}\in\mathbb{Z}^+$ *satisfying* $0\le V\le r_j, 0\le U\le s_j$ *for* $j=1,2,\cdots m$, $\mathfrak{D}_n,\mathfrak{E}_n,\mathfrak{E}_{nj},\mathfrak{D}_{nj}\in\mathbb{R}^+$, $\Lambda_n,\varepsilon_n,\Lambda_{nj},\varepsilon_{nj}\in\mathbb{C}\ (j=1,2,\cdots,r_j;n=1,2,\cdots,s_j)$, $A_{\mathfrak{Q}_k,\mathfrak{P}_k}^{k}$ *are real or complex numbers arbitrary constant for* $k=1,2,3,\cdots,s$ *and* $\rho_i>0$ *for* $i=1,2,\cdots,m$, *then the outcome shown below is accurate:*

$$\mathscr{D}_{0+}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi}\left(t^{\alpha-1}\prod_{j=1}^{s}S_{\mathfrak{Q}_j}^{\mathfrak{P}_j}[c_j t^{\lambda_j}]\right.$$
$$\left.\gamma\aleph_{r_j,s_j,\rho_j;m}^{U,V}\left[zt^{\mu}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]\right)(x)$$
$$=x^{\alpha+\varsigma+\varsigma'-\varpi-1}\sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}A_{\mathfrak{Q}_1,\mathfrak{P}_1}^{(1)}\cdots A_{\mathfrak{Q}_s,\mathfrak{P}_s}^{(s)}c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}(x)^{\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j}$$
$$\times\gamma\aleph_{r_j+3,s_j+3,\rho_j;m}^{U,V+3}\left[zx^{\mu}\left|\begin{matrix}(1-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1-\alpha-\varsigma-\varsigma'-\varkappa'+\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},(1-\varsigma-\varsigma'+\varpi-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\end{matrix}\right.\right.$$
$$\left.\begin{matrix}(1-\alpha-\varsigma+\varkappa-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(1-\alpha+\varkappa-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1-\varsigma-\varkappa'+\varpi-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right]. \tag{49}$$

Proof. Theorem 10 is proved in the same way as Theorem 9 with the same conditions.

The following corollary is obtained regarding the Saigo FIO [10] in light of the equation (16).

Corollary 11. *Let* ς, $\varkappa$, $\varpi, z, \alpha \in \mathbb{C}$ *and* $\Re(\varsigma) > 0, \mu > 0, \lambda_k > 0$ $(k = 1,2,3,\cdots,s)$,

$$\mu \max_{1\le j\le U}\left[\frac{-\Re(\varepsilon_j)}{\mathfrak{E}_j}\right] < \Re(\alpha) + \min[0, \Re(\varsigma-\varkappa), \Re(\varsigma-\varpi)].$$

Further the parameters r_j, $s_j, \mathfrak{P} \in \mathbb{Z}^+$ *satisfying* $0 \le V \le r_j, 0 \le U \le s_j$ *for* $j = 1,2,\cdots m$, $\mathfrak{D}_n, \mathfrak{E}_n, \mathfrak{E}_{nj}, \mathfrak{D}_{nj} \in \mathbb{R}^+$, $\Lambda_n, \varepsilon_n, \Lambda_{nj}, \varepsilon_{nj} \in \mathbb{C}$ $(j = 1,2,\cdots,r_j; n = 1,2,\cdots,s_j)$, $A^k_{\mathfrak{Q}_k,\mathfrak{P}_k}$ *are real or complex numbers arbitrary constant for* $k = 1,2,\cdots,s$ *and* $\rho_i > 0$ *for* $i = 1,2,\cdots,m$, *then the outcome shown below is accurate:*

$$\begin{aligned}
&\mathscr{D}^{\varsigma,\varkappa,\varpi}_{0+}\Bigg(t^{\alpha-1}\prod_{j=1}^{s} S^{\mathfrak{P}_j}_{\mathfrak{Q}_j}[c_j t^{\lambda_j}] \\
&{}^{\Gamma}\aleph^{U,V}_{r_j,s_j,\rho_j;m}\left[zt^{\mu}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\ (\varepsilon_n,\mathfrak{E}_n)_{1,U},[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]\Bigg)(x)\\
&= x^{\alpha+\varsigma+\varsigma'-\varpi-1}\sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}A^{(1)}_{\mathfrak{Q}_1,\mathfrak{P}_1}\cdots A^{(s)}_{\mathfrak{Q}_s,\mathfrak{P}_s}\,c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}\,(x)^{\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j}\\
&\times {}^{\Gamma}\aleph^{U,V+2}_{r_j+2,s_j+2,\rho_j;m}\left[zx^{\mu}\left|\begin{matrix}(1-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1-\alpha-\varsigma-\varkappa-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\\ (\varepsilon_n,\mathfrak{E}_n)_{1,U},(1-\alpha-\varkappa-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\end{matrix}\right.\right.\\
&\qquad\left.\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\ (1-\alpha-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right]. \qquad (50)
\end{aligned}$$

The same result can be obtained concerning Saigo fractional derivative operator for the lower incomplete ℵ-function.

Remark 8. By substituting $\varkappa = -\varsigma$ and $\varkappa = 0$ in Corollary 11, respectively, we can also get findings for the fractional derivative operators of R-L and E-K.

Theorem 12. *Let* ς, ς', $\varkappa$, $\varkappa'$, $\varpi, z, \alpha \in \mathbb{C}$ *and* $\Re(\varpi) > 0, \mu > 0, \lambda_k > 0$ $(k = 1,2,3,\cdots,s)$,

$$\mu \min_{1\le j\le V}\left[\frac{1-\Re(\Lambda_j)}{\mathfrak{D}_j}\right] + 1 > \Re(\alpha) - \min[0, \Re(\varpi-\varsigma-\varsigma'-V), \Re(-\varsigma'-\varkappa+\varpi), -\Re(\varkappa')].$$

Further the parameters r_j, $s_j, \mathfrak{P} \in \mathbb{Z}^+$ *satisfying* $0 \le V \le r_j, 0 \le U \le s_j$ *for* $j = 1,2,\cdots m$, $\mathfrak{D}_n, \mathfrak{E}_n, \mathfrak{E}_{nj}, \mathfrak{D}_{nj} \in \mathbb{R}^+$, $\Lambda_n, \varepsilon_n, \Lambda_{nj}, \varepsilon_{nj} \in \mathbb{C}$ $(j = 1,2,\cdots,r_j; n = 1,2,\cdots,s_j)$, $A^k_{\mathfrak{Q}_k,\mathfrak{P}_k}$ *are real or complex numbers arbitrary constant for*

$k = 1,2,\cdots,s$ *and* $\rho_i > 0$ *for* $i = 1,2,\cdots,m$*, then the outcome shown below is accurate:*

$$\mathscr{D}_{-}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi}\left(t^{\alpha-1}\prod_{j=1}^{s}S_{\mathfrak{Q}_j}^{\mathfrak{P}_j}[c_j t^{\lambda_j}]\right.$$
$$\left.{}^{\Gamma}\aleph_{r_j,s_j,\rho_j;m}^{U,V}\left[zt^{\mu}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]\right)(x)$$
$$=x^{\alpha+\varsigma+\varsigma'-\varpi-1}\sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}A_{\mathfrak{Q}_1,\mathfrak{P}_1}^{(1)}\cdots A_{\mathfrak{Q}_s,\mathfrak{P}_s}^{(s)}\,c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}\,(x)^{\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j}$$
$$\times{}^{\Gamma}\aleph_{r_j+3,s_j+3,\rho_j;m}^{U+3,V}\left[zx^{\mu}\left|\begin{matrix}(1-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1-\alpha-\varsigma'+\varkappa'-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},(1-\varsigma-\varsigma'+\varpi-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\end{matrix}\right.\right.$$
$$\left.\begin{matrix}(1-\alpha-\varsigma-\varsigma'-\varkappa+\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(1-\alpha+\varkappa'-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1-\varsigma'-\varkappa+\varpi-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right].\quad(51)$$

Proof. The LHS of equation (51) is:

$$T_4=\mathscr{D}_{-}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi}\left(t^{\alpha-1}\prod_{j=1}^{s}S_{\mathfrak{Q}_j}^{\mathfrak{P}_j}[c_j t^{\lambda_j}]\right.$$
$$\left.{}^{\Gamma}\aleph_{r_j,s_j,\rho_j;m}^{U,V}\left[zt^{\mu}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]\right)(x).\quad(52)$$

Replace the incomplete $\aleph$-function and Srivastava polynomial by equation (24) and (4) respectively and by reversing the summation order, we discover the subsequent form:

$$T_3=\sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}\times A_{\mathfrak{Q}_1,\mathfrak{P}_1}^{(1)}\cdots A_{\mathfrak{Q}_s,\mathfrak{P}_s}^{(s)}\,c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}$$
$$\times\frac{1}{2\pi\iota}\int_{\$}\Psi(q,\mathscr{Y})\,z^{-q}\left(\mathscr{D}_{-}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi}t^{-(-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j+\mu q+1)}\right)(x)dq,\quad(53)$$

where $\Psi(q,\mathscr{Y})$ is defined in equation (25).

Using equation (21) of Lemma 2, we discover the subsequent form:

$$T_4=\sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}\times A_{\mathfrak{Q}_1,\mathfrak{P}_1}^{(1)}\cdots A_{\mathfrak{Q}_s,\mathfrak{P}_s}^{(s)}\,c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}$$
$$\times\frac{1}{2\pi\iota}\int_{\$}x^{\alpha-\varsigma-\varsigma'+\varpi+\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j-1}\,\Psi(q,\mathscr{Y})\,(zx^{\mu})^{-q}\frac{\Gamma(1-\alpha+\varkappa'-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j+\mu q)}{\Gamma(1-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j+\mu q)}\times$$
$$\frac{\Gamma(1-\varsigma-\varsigma'+\varpi-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j+\mu q)\Gamma(1-\alpha-\varsigma'-\varkappa+\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j+\mu q)}{\Gamma(1-\alpha-\varsigma'+\varkappa'-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j+\mu q)\Gamma(1-\alpha-\varsigma-\varsigma'-\varkappa+\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j+\mu q)}dq.\quad(54)$$

Finally, after some adjustment of terms, we obtain RHS of equation (51).

Theorem 13. *Let* $\varsigma,\varsigma',\varkappa,\varkappa',\varpi,z,\alpha\in\mathbb{C}$ *and* $\Re(\varpi)>0,\mu>0,\lambda_k>0\,(k=1,2,3,\cdots,s)$,

$$\mu\min_{1\leq j\leq V}\left[\frac{1-\Re(\Lambda_j)}{\mathfrak{D}_j}\right]+1>\Re(\alpha)-\min[0,\Re(\varpi-\varsigma-\varsigma'-V),\Re(-\varsigma'-\varkappa+\varpi),-\Re(\varkappa')].$$

Further the parameters r_j, $s_j, \mathfrak{P} \in \mathbb{Z}^+$ satisfying $0 \le V \le r_j, 0 \le U \le s_j$ for $j = 1,2,\cdots m$, $\mathfrak{D}_n, \mathfrak{E}_n, \mathfrak{E}_{nj}, \mathfrak{D}_{nj} \in \mathbb{R}^+$, $\Lambda_n, \varepsilon_n, \Lambda_{nj}, \varepsilon_{nj} \in \mathbb{C}$ $(j = 1,2,\cdots,r_j; n = 1,2,\cdots,s_j)$, $A^k_{\mathfrak{Q}_k,\mathfrak{P}_k}$ are real or complex numbers arbitrary constant for $k = 1,2,\cdots,s$ and $\rho_i > 0$ for $i = 1,2,\cdots,m$, then the outcome shown below is accurate:

$$\mathscr{D}_{-}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi}\left(t^{\alpha-1}\prod_{j=1}^{s}S_{\mathfrak{Q}_j}^{\mathfrak{P}_j}[c_j t^{\lambda_j}]\right.$$
$$\left.{}^{\gamma}\aleph^{U,V}_{r_j,s_j,\rho_j;m}\left[zt^{\mu}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]\right)(x)$$
$$= x^{\alpha+\varsigma+\varsigma'-\varpi-1}\sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}A^{(1)}_{\mathfrak{Q}_1,\mathfrak{P}_1}\cdots A^{(s)}_{\mathfrak{Q}_s,\mathfrak{P}_s}\,c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}\,(x)^{\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j}$$
$$\times{}^{\gamma}\aleph^{U+3,V}_{r_j+3,s_j+3,\rho_j;m}\left[zx^{\mu}\left|\begin{matrix}(1-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1-\alpha-\varsigma'+\varkappa'-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},(1-\varsigma-\varsigma'+\varpi-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\end{matrix}\right.\right.$$
$$\left.\begin{matrix}(1-\alpha-\varsigma-\varsigma'-\varkappa+\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(1-\alpha+\varkappa'-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1-\varsigma'-\varkappa+\varpi-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right]. \tag{55}$$

Proof. Theorem 13 is proved in the same way as Theorem 12 with the same conditions.

The following corollary is obtained regarding the Saigo fractional derivative operator [10] in light of the equation (16).

Corollary 14. *Let $\varsigma, \varsigma', \varkappa, \varkappa', \varpi, z, \alpha \in \mathbb{C}$ and $\Re(\varpi) > 0, \mu > 0, \lambda_k > 0$ $(k = 1,2,3,\cdots,s)$,*

$$\mu\min_{1\le j\le V}\left[\frac{1-\Re(\Lambda_j)}{\mathfrak{D}_j}\right]+1>\Re(\alpha)-\min[0,\Re(\varpi-\varsigma-\varsigma'-V),\Re(-\varsigma'-\varkappa+\varpi),-\Re(\varkappa')].$$

Further the parameters r_j, $s_j, \mathfrak{P} \in \mathbb{Z}^+$ satisfying $0 \le V \le r_j, 0 \le U \le s_j$ for $j = 1,2,\cdots m$, $\mathfrak{D}_n, \mathfrak{E}_n, \mathfrak{E}_{nj}, \mathfrak{D}_{nj} \in \mathbb{R}^+$, $\Lambda_n, \varepsilon_n, \Lambda_{nj}, \varepsilon_{nj} \in \mathbb{C}$ $(j = 1,2,\cdots,r_j; n = 1,2,\cdots,s_j)$, $A^k_{\mathfrak{Q}_k,\mathfrak{P}_k}$ are real or complex numbers arbitrary constant for $k = 1,2,\cdots,s$ and $\rho_i > 0$ for $i = 1,2,\cdots,m$, then the following result holds:

$$\mathscr{D}_{-}^{\varsigma,\varkappa,\varpi}\left(t^{\alpha-1}\prod_{j=1}^{s}S_{\mathfrak{Q}_j}^{\mathfrak{P}_j}[c_j t^{\lambda_j}]\right.$$
$$\left.{}^{\Gamma}\aleph^{U,V}_{r_j,s_j,\rho_j;m}\left[zt^{\mu}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right.\right]\right)(x)$$
$$= x^{\alpha+\varkappa-1}\sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}A^{(1)}_{\mathfrak{Q}_1,\mathfrak{P}_1}\cdots A^{(s)}_{\mathfrak{Q}_s,\mathfrak{P}_s}\,c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}\,(x)^{\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j}$$
$$\times{}^{\Gamma}\aleph^{U+2,V}_{r_j+2,s_j+2,\rho_j;m}\left[zx^{\mu}\left|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(1-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},(1-\varkappa-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\end{matrix}\right.\right.$$
$$\left.\begin{matrix}(1-\alpha-\varkappa+\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(\Lambda_n,\mathfrak{D}_n)_{2,V},[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(1+\varsigma+\varpi-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right]. \tag{56}$$

The same result can be obtained regarding Saigo fractional derivative operator for the lower incomplete $\aleph$-function.

Remark 9. By substituting $\varkappa = -\varsigma$ and $\varkappa = 0$ in Corollary 14, respectively, we can also get findings for the fractional derivative operators of R-L and E-K.

5 Special cases and applications

This section focuses on a few fascinating unique cases of Theorem 3. For other theorems, it is simple for us to obtain comparable findings.

(i) On setting $\mathscr{Y}=0$, in Theorem 3 and in consideration of equation (26), then incomplete $\aleph$-function reduce to the $\aleph$-function proposed by Sudland [26, 27] and we reach the following conclusion:

$$\mathscr{I}_{0+}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi}\left(t^{\alpha-1}\prod_{j=1}^{s}S_{\mathfrak{Q}_j}^{\mathfrak{P}_j}[c_j t^{\lambda_j}]\,\aleph_{r_j,s_j,\rho_j;m}^{U,V}\left[zt^{\mu}\middle|\begin{matrix}(\Lambda_n,\mathfrak{D}_n)_{1,V},\,[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},\,[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right]\right)(x)$$
$$=x^{\alpha-\varsigma-\varsigma'+\varpi-1}\sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}A^{(1)}_{\mathfrak{Q}_1,\mathfrak{P}_1}\cdots A^{(s)}_{\mathfrak{Q}_s,\mathfrak{P}_s}\,c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}\,(x)^{\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j}$$
$$\times\,\aleph_{r_j+3,s_j+3,\rho_j;m}^{U,V+3}\left[zx^{\mu}\middle|\begin{matrix}(1-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1-\alpha+\varsigma+\varsigma'+\varkappa-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},\,(1-\alpha+\varsigma+\varsigma'-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\\(1-\alpha+\varsigma'-\varkappa'-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\,(\Lambda_n,\mathfrak{D}_n)_{1,V},\,[\rho_n(\Lambda_{nj},\mathfrak{D}_{nj})]_{V+1,r_j}\\(1-\alpha-\varkappa'-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\,(1+\varsigma'+\varkappa-\varpi-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\,[\rho_n(\varepsilon_{nj},\mathfrak{E}_{nj})]_{U+1,s_j}\end{matrix}\right].\tag{57}$$

(ii) Again, setting $\rho_j=1$ in Theorem 3 and in consideration of equation (28), then incomplete $\aleph$-function reduces to the Incomplete I-function suggested by Bansal and Kumar [28] and we reach the following conclusion:

$$\mathscr{I}_{0+}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi}\left(t^{\alpha-1}\prod_{j=1}^{s}S_{\mathfrak{Q}_j}^{\mathfrak{P}_j}[c_j t^{\lambda_j}]\,{}^{\Gamma}I_{r_j,s_j;m}^{U,V}\left[zt^{\mu}\middle|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},\,(\Lambda_{nj},\mathfrak{D}_{nj})_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},\,(\varepsilon_{nj},\mathfrak{E}_{nj})_{U+1,s_j}\end{matrix}\right]\right)(x)$$
$$=x^{\alpha-\varsigma-\varsigma'+\varpi-1}\sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}A^{(1)}_{\mathfrak{Q}_1,\mathfrak{P}_1}\cdots A^{(s)}_{\mathfrak{Q}_s,\mathfrak{P}_s}\,c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}\,(x)^{\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j}$$
$$\times\,{}^{\Gamma}I_{r_j+3,s_j+3;m}^{U,V+3}\left[zx^{\mu}\middle|\begin{matrix}(1-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1-\alpha+\varsigma+\varsigma'+\varkappa-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},\,(1-\alpha+\varsigma+\varsigma'-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\\(1-\alpha+\varsigma'-\varkappa'-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\,(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,V},\,(\Lambda_{nj},\mathfrak{D}_{nj})_{V+1,r_j}\\(1-\alpha-\varkappa'-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\,(1+\varsigma'+\varkappa-\varpi-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\,(\varepsilon_{nj},\mathfrak{E}_{nj})_{U+1,s_j}\end{matrix}\right].\tag{58}$$

(iii) Next, setting $\mathscr{Y}=0$ and $\rho_j=1$ in Theorem 3 and in consideration of equation (29), then incomplete $\aleph$-function reduce to the I-function suggested by Saxena [29] and we reach the following conclusion:

$$\mathscr{I}_{0+}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi}\left(t^{\alpha-1}\prod_{j=1}^{s}S_{\mathfrak{Q}_j}^{\mathfrak{P}_j}[c_j t^{\lambda_j}]\,I_{r_j,s_j;m}^{U,V}\left[zt^{\mu}\middle|\begin{matrix}(\Lambda_n,\mathfrak{D}_n)_{1,V},\,(\Lambda_{nj},\mathfrak{D}_{nj})_{V+1,r_j}\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},\,(\varepsilon_{nj},\mathfrak{E}_{nj})_{U+1,s_j}\end{matrix}\right]\right)(x)$$
$$=x^{\alpha-\varsigma-\varsigma'+\varpi-1}\sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}A^{(1)}_{\mathfrak{Q}_1,\mathfrak{P}_1}\cdots A^{(s)}_{\mathfrak{Q}_s,\mathfrak{P}_s}\,c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}\,(x)^{\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j}$$
$$\times\,I_{r_j+3,s_j+3;m}^{U,V+3}\left[zx^{\mu}\middle|\begin{matrix}(1-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1-\alpha+\varsigma+\varsigma'+\varkappa-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\\(\varepsilon_n,\mathfrak{E}_n)_{1,U},\,(1-\alpha+\varsigma+\varsigma'-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\\(1-\alpha+\varsigma'-\varkappa'-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\,(\Lambda_n,\mathfrak{D}_n)_{1,V},\,(\Lambda_{nj},\mathfrak{D}_{nj})_{V+1,r_j}\\(1-\alpha-\varkappa'-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\,(1+\varsigma'+\varkappa-\varpi-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\,(\varepsilon_{nj},\mathfrak{E}_{nj})_{U+1,s_j}\end{matrix}\right].\tag{59}$$

(iv) Further setting $\rho_j = 1$ and $m = 1$ in Theorem 3 and in consideration of equation (31), then incomplete ℵ-function reduce to the incomplete H-function suggested by Srivastava [32] and we reach the following conclusion:

$$\mathscr{I}_{0+}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi}\left(t^{\alpha-1}\prod_{j=1}^{s}S_{\mathfrak{Q}_j}^{\mathfrak{P}_j}[c_j t^{\lambda_j}]\Gamma_{r,s}^{U,V}\left[zt^{\mu}\middle|\begin{matrix}(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,r}\\(\varepsilon_n,\mathfrak{E}_n)_{1,s}\end{matrix}\right]\right)(x)$$
$$= x^{\alpha-\varsigma-\varsigma'+\varpi-1}\sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}A_{\mathfrak{Q}_1,\mathfrak{P}_1}^{(1)}\cdots A_{\mathfrak{Q}_s,\mathfrak{P}_s}^{(s)}\,c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}\,(x)^{\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j}$$
$$\times\Gamma_{r+3,s+3}^{U,V+3}\left[zx^{\mu}\middle|\begin{matrix}(1-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1-\alpha+\varsigma+\varsigma'+\varkappa-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\\(\varepsilon_n,\mathfrak{E}_n)_{1,s},(1-\alpha+\varsigma+\varsigma'-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\end{matrix}\right.$$
$$\left.\begin{matrix}(1-\alpha+\varsigma'-\varkappa'-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(\Lambda_1,\mathfrak{D}_1:\mathscr{Y}),(\Lambda_n,\mathfrak{D}_n)_{2,r}\\(1-\alpha-\varkappa'-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1+\varsigma'+\varkappa-\varpi-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu)\end{matrix}\right]. \tag{60}$$

(v) Next, setting $\mathscr{Y}=0$, $\rho_j=1$, and $m=1$ in Theorem 3 and in consideration of equation (32), then incomplete ℵ-function reduce to the H-function suggested by Srivastava [33] and we reach the following conclusion:

$$\mathscr{I}_{0+}^{\varsigma,\varsigma',\varkappa,\varkappa',\varpi}\left(t^{\alpha-1}\prod_{j=1}^{s}S_{\mathfrak{Q}_j}^{\mathfrak{P}_j}[c_j t^{\lambda_j}]H_{r,s}^{U,V}\left[zt^{\mu}\middle|\begin{matrix}(\Lambda_n,\mathfrak{D}_n)_{1,r}\\(\varepsilon_n,\mathfrak{E}_n)_{1,s}\end{matrix}\right]\right)(x)$$
$$= x^{\alpha-\varsigma-\varsigma'+\varpi-1}\sum_{\mathfrak{k}_1=0}^{[\mathfrak{Q}_1/\mathfrak{P}_1]}\cdots\sum_{\mathfrak{k}_s=0}^{[\mathfrak{Q}_s/\mathfrak{P}_s]}\frac{(-\mathfrak{Q}_1)_{\mathfrak{P}_1\mathfrak{k}_1}\cdots(-\mathfrak{Q}_s)_{\mathfrak{P}_s\mathfrak{k}_s}}{\mathfrak{k}_1!\cdots\mathfrak{k}_s!}A_{\mathfrak{Q}_1,\mathfrak{P}_1}^{(1)}\cdots A_{\mathfrak{Q}_s,\mathfrak{P}_s}^{(s)}\,c_1^{\mathfrak{k}_1}\cdots c_s^{\mathfrak{k}_s}\,(x)^{\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j}$$
$$\times H_{r+3,s+3}^{U,V+3}\left[zx^{\mu}\middle|\begin{matrix}(1-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1-\alpha+\varsigma+\varsigma'+\varkappa-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\\(\varepsilon_n,\mathfrak{E}_n)_{1,s},(1-\alpha+\varsigma+\varsigma'-\varpi-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),\end{matrix}\right.$$
$$\left.\begin{matrix}(1-\alpha+\varsigma'-\varkappa'-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(\Lambda_n,\mathfrak{D}_n)_{1,r}\\(1-\alpha-\varkappa'-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu),(1+\varsigma'+\varkappa-\varpi-\alpha-\sum_{j=1}^{s}\lambda_j\mathfrak{k}_j,\mu)\end{matrix}\right]. \tag{61}$$

Remark 10.

- The known results provided by Saxena and Saigo [35] are simple to be achieved if the generic class of polynomials $S_L^{h_1,\cdots,h_s}$ is restricted to unity and the incomplete ℵ-function is reduced to Fox's H-function.
- If we set incomplete ℵ-function to ℵ-function, then we may easily achieve the results that Saxena and Kumar [36] have already provided.
- The known results provided by Saxena and Ram [37] are simple to be achieved if the generic class of polynomials $S_L^{h_1,\cdots,h_s}$ is restricted to unity and the incomplete ℵ-function is reduced to ℵ- function.
- Theorems 4 and Theorem 5 provided by Bansal et al. [24] are simply obtained if we set the generic class of polynomials $S_L^{h_1,\cdots,h_s}$ in Theorem 3 and Theorem 6 to unity.

6 Conclusion

In the current paper, we looked into a variety of incomplete ℵ-function based FC image formulae as well as the generic class of polynomials connected to the MSM operators. The incomplete ℵ-functions are the generalized form of various other special functions. Also, Srivastava polynomial generalize various other polynomials like: Hermite polynomial, Jacobi polynomial, Laguerre polynomial, Gegenbauer polynomial, Legendre polynomial, Tchebycheff polynomial, Gould-Hopper Polynomial and several other polynomials. Additionally, the MSM fractional operators generalize Saigo, R-L and E-K FC operators. One may get a variety of image formulae that include a class of special functions by taking the mentioned fact into consideration [23, 38–40] as limiting instances of the primary outcomes.

7 Declarations

7.1 Conflict of interest:

Authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

7.2 Funding:

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7.3 Author's contribution:

N.-Conceptualization. S.B.-Data Curation, Writing-Review Editing. S.D.P.-Software, Writing-Original Draft. K.S.N.-Supervisor. S.R.M.-Methodology. All authors read and approved the final submitted version of this manuscript. All authors contributed equally to the manuscript and approved the final manuscript.

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7.5 Data availability statement:

All data that support the findings of this study are included within the article.

7.6 Using of AI tools:

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

References

[1] Bagley R.L., Torvik P.J., A theoretical basis for the application of fractional calculus to viscoelasticity, Journal of Rheology, 27(3), 201–210, 1983.
[2] Fellah Z.E.A., Depollier C., Fellah M., Application of fractional calculus to the sound waves propagation in rigid porous materials: validation via ultrasonic measurements, Acta Acustica United with Acustica, 88(1), 34–39, 2002.
[3] Rossikhin Y.A., Shitikova M.V., Application of fractional calculus for dynamic problems of solid mechanics: novel trends and recent results, Applied Mechanics Reviews, 63(1), 010801, 2010.
[4] Sun Y., Zheng C., Fractional-order modelling of state-dependent non-associated behaviour of soil without using state variable and plastic potential, Advances in Difference Equations, 2019(1), 83, 2019.
[5] Ata E., Kıymaz İ.O., Generalized gamma, beta and hypergeometric functions defined by wright function and applications to fractional differential equations, Cumhuriyet Science Journal, 43(4), 684–695, 2022.

[6] Magin R.L., Fractional calculus in bioengineering: Part 2, Critical Reviews in Biomedical Engineering, 32(2), 105–194, 2004.
[7] Ata E., M-Lauricella hypergeometric functions: integral representations and solutions of fractional differential equations, Communications Faculty of Sciences University of Ankara Series, A1 Mathematics and Statistics, 72(2), 512–529, 2023.
[8] Srivastava H.M., Saxena R.K., Operators of fractional integration and their applications, Applied Mathematics and Computation, 118(1), 1–52, 2001.
[9] Debnath L., Bhatta D., Integral Transforms and Their Applications, (Third Ed.), Chapman and Hall (CRC Press), Taylor and Francis Group, London and New York, USA, 2015.
[10] Saigo M., A remark on integral operators involving the Gauss hypergeometric functions, Mathematical Reports of College of General Education, Kyushu University, 11(2), 135–143, 1978.
[11] Samko S.G., Kilbas A.A., Marichev O.I., Fractional Integrals and Derivatives: Theory and Applications; Gordon and Breach: Yverdon, Switzerland, 1993.
[12] Miller K.S., Ross B., An Introduction to Fractional Calculus and Fractional Differential Equations, Wiley, New York, USA, 1993.
[13] Jangid K., Bhatter S., Meena S., Baleanu D., Al-Qurashi M., Purohit S.D., Some fractional calculus findings associated with the incomplete I-functions, Advances in Difference Equations, 2020(1), 265, 2020.
[14] Love E.R., Some integral equations involving hypergeometric functions, Proceedings of the Edinburgh Mathematical Society, 15(3), 169–198, 1967.
[15] Ram J., Kumar D., Generalized fractional integration of the ℵ-function, Journal of Rajasthan Academy of Physical Sciences, 10(4), 373–382, 2011.
[16] Chaudhry M.A., Zubair S.M., Generalized incomplete gamma functions with applications, Journal of Computational and Applied Mathematics, 55(1), 99-124, 1994.
[17] Srivastava H.M., Singh N.P., The integration of certain products of the multivariable H-function with a general class of polynomials, Rendiconti del Circolo Matematico di Palermo, 32(2), 157–187, 1983.
[18] Bhatter S., Nishant, Suthar D.L., Purohit S.D., Boros integral involving the product of family of polynomials and the incomplete I-function, Journal of Computational Analysis and Applications, 31(3), 400–412, 2023.
[19] Marichev O.I., Volterra equation of Mellin convolution type with a Horn function in the kernel, Izvestiya Akademii Nauk Belarusi. Seriya Fiziko-Matematicheskikh Nauk, 1, 128–129, 1974.
[20] Saigo M., Maeda N., More Generalization of Fractional Calculus, In Transform Methods and Special Functions, Varna'96 Edited by Rusev P., Dimovski I., Kiryakova V., (Proc. Second Internat. Workshop), Science Culture Technology Publishing: Singapore, 386–400, 1998.
[21] Baleanu D., Kumar D., Purohit S.D., Generalized fractional integrals of product of two H-functions and a general class of polynomials, International Journal of Computer Mathematics, 93(8), 1320–1329, 2016.
[22] Purohit S.D., Suthar D.L., Kalla S.L., Marichev-Saigo-Maeda fractional integration operators of the Bassel functions, Le Matematiche, 67(1), 21–32, 2012.
[23] Kilbas A.A., Srivastava H.M., Trujillo J.J., Theory and Applications of Fractional Differential Equations, North-Holland Mathematical Studies, Elsevier, New York, USA, (1st Ed.) 204, USA, 2006.
[24] Bansal M.K., Kumar D., Nisar K.S., Singh J., Certain fractional calculus and integral transform results of incomplete ℵ-functions with applications, Mathematical Methods in the Applied Sciences, 43(8), 5602–5614, 2020.
[25] Bhatter S., Nishant, Shyamsunder, Mathematical model on the effects of environmental pollution on biological populations, Advances in Mathematical Modelling, Applied Analysis and Computation, ICMMAAC 2022, Springer, 666, 488–496, 2023.
[26] Südland N., Baumann G., Nonnenmacher T.F., Fractional driftless Fokker-Planck equation with power law diffusion coefficients, Computer Algebra in Scientific Computing CASC 2001, Proceedings of the Fourth International Workshop on Computer Algebra in Scientific Computing, Konstanz, Springer Berlin Heidelberg, Germany, 513–528, 2001.
[27] Südland N., Baumann G., Nonnenmacher T.F., Open problem: who knows about the ℵ-function?, Fractional Calculus and Applied Analysis, 1(4), 401–402, 1998.
[28] Bansal M.K., Kumar D., On the integral operators pertaining to a family of incomplete I-functions, AIMS Mathematics, 5(2), 1247–1259, 2020.
[29] Saxena V.P., Formal solution of certain new pair of dual integral equations involving H-functions, Proceedings of the National Academy of Sciences, India Section A: Physical Sciences, 52, 366–375, 1982.
[30] Jangid K., Bhatter S., Meena S., Purohit S.D., Certain classes of the incomplete I-functions and their properties, Discontinuity Nonlinearity and Complexity, 12(2), 437–454, 2023.
[31] Jangid K., Purohit S.D., Agarwal R., Agarwal R.P., On the generalization of fractional kinetic equation comprising incomplete H-function, Kragujevac Journal of Mathematics, 47(5), 701–712, 2023.
[32] Srivastava H.M., Chaudhry M.A., Agarwal R.P., The incomplete Pochhammer symbols and their applications to hypergeometric and related functions, Integral Transforms and Special Functions, 23(9), 659–683, 2012.
[33] Srivastava H.M., Gupta K.C., Goyal S.P., The H-functions of One and Two Variables with Applications, New Delhi

and Madras, South Asian Publishers, India, 1982.
[34] Srivastava H.M., Saxena R.K., Parmar R.K., Some families of the incomplete H-functions and the incomplete $\overline{H}$-functions and associated integral transforms and operators of fractional calculus with applications, Russian Journal of Mathematical Physics, 25(1), 116–138, 2018.
[35] Saxena R.K., Saigo M., Generalized fractional calculus of the H-function associated with the Appell function F_3, Journal of Fractional Calculus, 19, 89–104, 2001.
[36] Saxena R.K., Generalized fractional calculus of the $\aleph$-function involving a general class of polynomials, Acta Mathematica Scientia, 35(5), 1095–1110, 2015.
[37] Saxena R.K., Ram J., Kumar D., Generalized fractional differentiation of the $\aleph$-function associated with the Appell function F_3, Acta Ciencia Indica, 38(4), 781–792, 2012.
[38] Ata E., Kıymaz İ.O., A study on certain properties of generalized special functions defined by Fox-Wright function, Applied Mathematics and Nonlinear Sciences, 5(1), 147–162, 2020.
[39] Chaudhry M.A., Qadir A., Srivastava H.M., Paris R.B., Extended hypergeometric and confluent hypergeometric functions, Applied Mathematics and Computation, 159(2), 589–602, 2004.
[40] Srivastava H.M., Agarwal P., Jain S., Generating functions for the generalized Gauss hypergeometric functions, Applied Mathematics and Computation, 247, 348–352, 2014.