

Auxetic Materials – A Review

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Auxetic materials are endowed with a behavior that contradicts common sense, when subjected to an axial tensile load they increase their transverse dimension. In case of a compression load, they reduce their transverse dimension. Consequently, these materials have a negative Poisson's ratio in such direction. This paper reviews research related to these materials. It presents the theories that explain their deformation behavior and reveals the important role represented by the internal structure. Their mechanical properties are explored and some potential applications for these materials are shown.

Keywords: *auxetics; Poisson's ratio; elasticity; deformation*

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1. Introduction

The Poisson's ratio of a material is a dimensionless constant that depends on the direction of an applied load, and describes the ratio of negative transverse strain to the longitudinal strain of a body submitted to a tensile load [1]. It provides a universal way to compare the structural performance of real homogeneous and non-homogeneous materials [2]. This elastic constant was implicitly assumed to be positive [3], as common sense dictated that no isotropic material in nature had a value of Poisson's ratio less than zero [4]. However, there are materials that present an inverse behavior. These materials expand their transverse dimension when submitted to an axial tensile strength and decrease it when compressed [5]. This way, they have a negative Poisson's ratio. The materials that reveal this behavior have been called anti-rubber [6] and dilational materials [7], but it was Ken Evans who coined the currently accepted term: "auxetics" [8]. This name, derived from the Greek word *auxetikos* ($\alpha\nu\chi\eta\tau\iota\kappa\omicron\varsigma$), means "that which tends to increase" [9].

This kind of behavior does not contradict the classical theory of elasticity [10, 11]. In the theory, for isotropic 3D materials the Poisson's ratio can assume values between -1 and 0.5 [12] while for

isotropic 2D materials it can assume values from -1 to 1 [10]. The violation of these limits gives rise to instability [13].

In isotropic systems, the Poisson's ratio is independent of the direction, but in the case of anisotropic materials the determination of this ratio depends on the direction of the stretch [14] and the other transverse directions [15]. There are materials that reveal an auxetic behavior in some directions and non-auxetic behavior in the others [16] (for example in α -Cristobalite [17]). These kinds of materials are known as partial auxetics [18, 19]. Contradicting common sense, partial auxetics are quite common, as 69 % of cubic elemental metals present auxetic behavior in at least one direction [20]. This interaction between the different directions of deformations may generate interesting values of Poisson's ratio that exceed largely the presented isotropic values [21, 22] for orthotropic and anisotropic materials [23].

Even though the existence of auxetic materials has been admitted for more than 150 years [24], only a few examples have been found in nature. In 1882 the case of *iron pyrite monocrystals* was reported by experiments on the twisting and bending of mineral rods [25]. This was the first study that proved the existence of this kind of material in nature. It was estimated that its Poisson's ratio was about $-1/7$ [26].

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Since then, other cases, such as *polymorphic silicicones* [27], *zeolites* [28] and *silicates* [29] have been considered. Examples of auxetic behavior in biological tissues were also found. There are some classical examples such as cat skin [30], cow teat skin [31] and cancellous bone [32]. Additional studies suggest that inorganic and biological fibrous materials in general present auxetic behavior [33].

Due to the shortage of auxetics in nature and the difficulty in attributing them a specific application, there was an effort to synthesize them [34]. This objective was completed for the first time in 1987, by manufacturing the first auxetic foam [35]. This was possible by working out the material internal structure, considering the way that it deforms when subjected to a load [36].

2. Structures

The approach to the manufacturing of auxetic materials, considering not only the base material, but the internal structure and deformation mechanism [37], allowed the expansion of the scale in which this behavior occurs. The control of the material structure made possible to tailor the material properties [38]. This way, it became possible to elaborate auxetic macrostructures [39].

Using these structural models, many theories have been developed to explain the behavior of these materials.

At this moment, there are some accepted structural deformation models, like *reentrant structures* [40, 41], the *rotating rigid and semi-rigid deformation model* [42] and *chiral structures* [43–47].

Reentrant structures are formed by hexagonal face cells, which have the edges protruding outwardly. These kinds of structures are represented in Fig. 1.

In the case of a uniaxial tensile load, the reentrant edges are subjected to bending and pulling simultaneously [49]. The consequence of this deformation is the simultaneous expansion of the cell faces, increasing cellular volume. As a result, the dimensions of the cell increase with the tensile de-

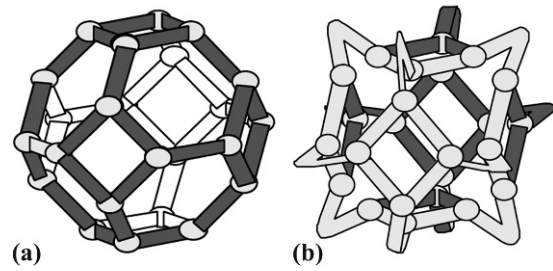


Fig. 1. A conventional cell (a) and an ideal auxetic cell (b) [48].

formation and the Poisson's ratio of the structure is negative.

In Fig. 2, a two-dimensional reentrant auxetic cell subjected to a tensile load is presented. As can be seen, the cell ribs tend to open, forcing the increase of the cell's area. This confirms the auxetic behavior of these structures.

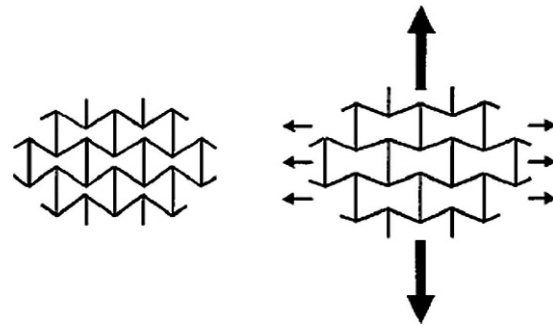


Fig. 2. Illustration of auxetic behavior on reentrant structures [49].

However, the auxetic behavior of these structures is more complex than the initial geometrical models predicted. It is known that the negative Poisson's ratio of these structures depends not only on the reentrant geometry, but also on the simultaneous flexure, hinging and stretching of the cell walls [50].

The rigid and semi-rigid rotation model is composed of a system of rigid geometry, connected by semi-rigid hinges in its corners. The layout of these structures is made in such way that a tensile deformation generates a bidirectional expansion [49], as shown in Fig. 3. Being subjected, for example to a tensile load, the hinges in the corners rotate, forcing the structure to unfold on itself.

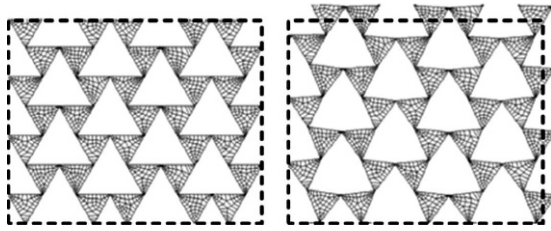
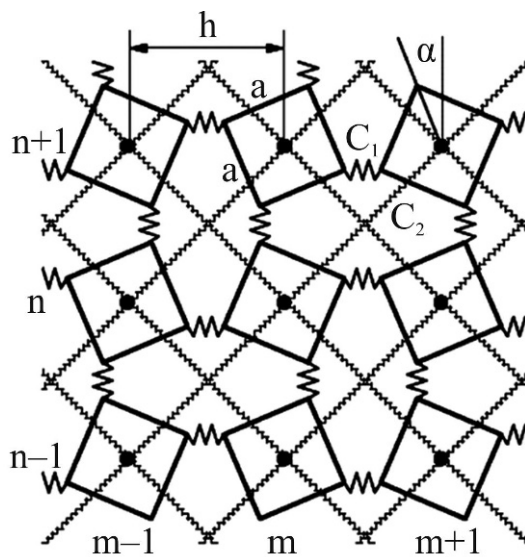
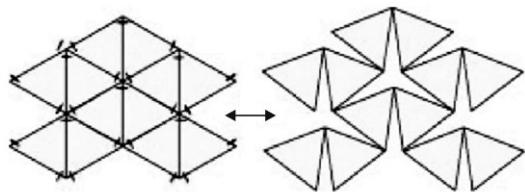


Fig. 3. Deformation behavior in rigid and semi-rigid rotating units [51].

This kind of internal structure can be obtained in numerous geometries, including rectangles [42], squares [52, 53], triangles [36, 51], and others [23, 37]. In Fig. 4 some examples of rigid units models that exhibit auxetic behavior are shown.



(a)



(b)

Fig. 4. Square shaped (a) and triangular (b) rigid unit models [36, 52].

The referred examples concern two-dimensional structures of rigid rotating units, however, recent studies demonstrate some attempts to transform them into three-dimensional structures, such as the one represented in Fig. 5 [54].

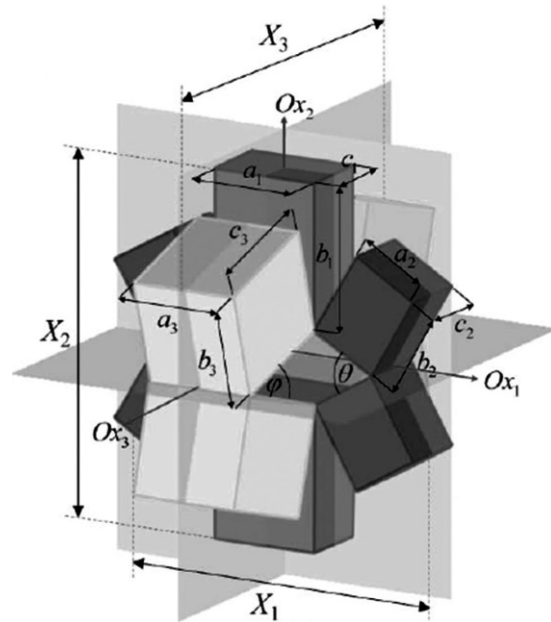


Fig. 5. Three-dimensional rotating cube structure [54].

The main characteristics of chiral structures is that they do not have a symmetric reflection, only rotational reflection [55]. Fundamentally, they are composed of a central node, connected by ribs, whose geometry can vary [49]. These two elements are joined by an almost tangential contact of the rib with the external face of the node. These structures are represented on a two-dimensional plane and are isotropic, (see Fig. 6). Their Poisson's ratio is close to -1 [56].

Due to their geometry, when chiral structures are submitted to stress, they have a particular deformation behavior. Applying a compressive or tensile load, each one of the individual cells suffers a torsion effect. In this way, the central node of every cell will rotate [55]. This rotation makes the cells twist and untwist, generating a contraction or expansion behavior in the whole structure. In Fig. 7, a chiral structure submitted to a uniaxial compressive load is presented. Considering the generated deformation, its auxetic behavior can be confirmed.

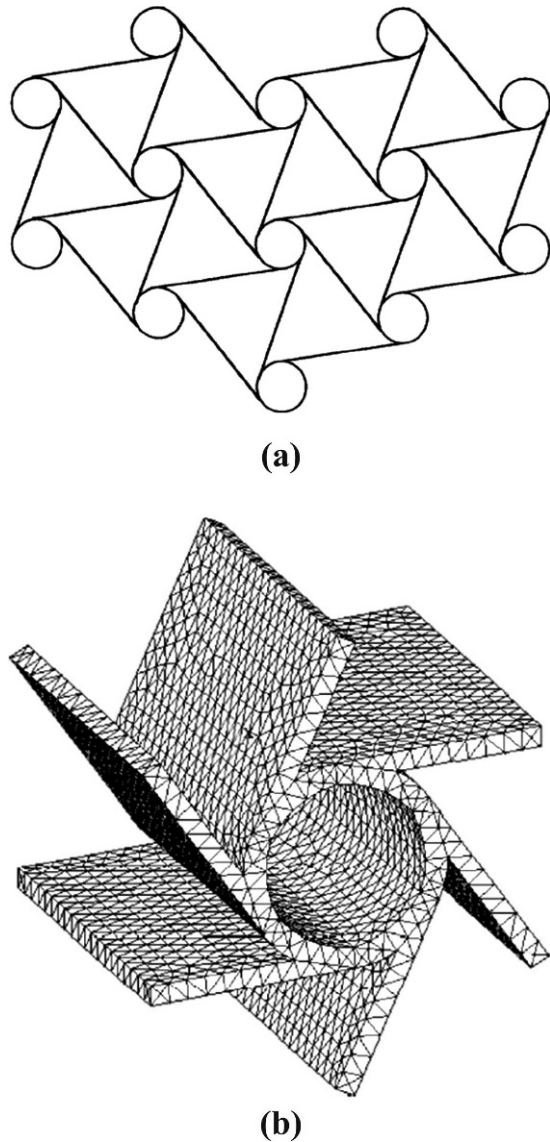


Fig. 6. Chiral structure (a) [56] and an individual chiral cell (b) [57].

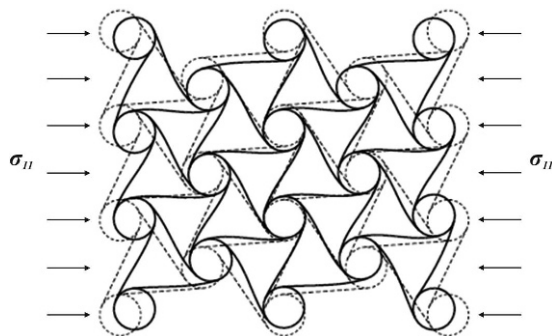


Fig. 7. Confirmation of auxetic behavior on a chiral structure [58].

3. Properties

Auxetic materials are characterized by a peculiar behavior and, as a consequence, they have peculiar and rare mechanical properties [59]. Exploring these materials, one can find enhanced properties [60] that contradict common sense, when compared to the characteristics of so called “regular” materials. Some of these properties are presented in the following sections.

3.1. Resistance to indentation

When a non-auxetic material is subjected to indentation, the load applied by the indenter locally compresses the material. To compensate this localized pressure, the material is spreading in the direction perpendicular to the applied load [61] (Fig. 8a).

However, when an indentation occurs in an isotropic auxetic material, a local contraction is observed. There is a flow of material that accumulates under the indenter (Fig. 8b), and an area of denser material with higher resistance to indentation is created [9]. In this way, auxetic materials have an improved indentation resistance, when compared to conventional materials [62, 63].

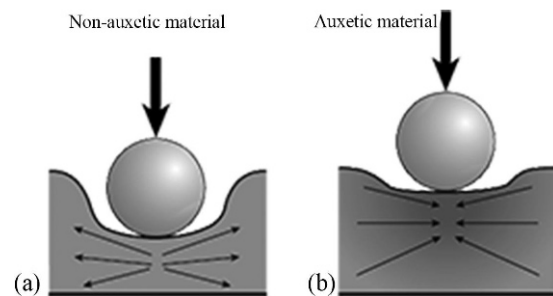


Fig. 8. Indentation behavior in non-auxetic (a) and auxetic (b) materials [64].

The increase in indentation resistance can be justified by the theory of elasticity. The indentation resistance is associated to the material hardness (H). This property is correlated to the Poisson’s ratio by equation 1 [65]:

$$H \propto \left[\frac{E}{(1 - \nu^2)} \right]^\gamma \quad (1)$$

where E is the Young’s modulus, ν is the Poisson’s ratio of the base materials and γ is the constant that assumes the value 1 or 2/3 in the case of uniform pressure distribution or hertzian indentation, respectively.

Analyzing equation 1, it can be inferred that for 3D isotropic materials, when the Poisson’s ratio decreases to the extreme values near -1 , the hardness of the material tends to infinity [66]. As the upper limit of Poisson’s ratio for 3D isotropic solids is 0.5, the observed values are considerably lower. However, the upper limit of the Poisson’s ratio for 2D isotropic systems is 1 [67, 68]. Thus, the materials with such positive Poisson’s ratio values can also have infinite hardness values.

3.2. Shear resistance

For similar situations, auxetic materials are more resistant to shear forces, than “regular” materials [65]. The classical theory of elasticity for 3D isotropic solids implies that the elastic behavior of a body can be described by two of four constants: the Young’s modulus (E), the shear modulus (G), the bulk modulus (K) and the Poisson’s ratio (ν) [69]. In 3D, the relationship between these constants is given by equations 2 and 3 [70]:

$$G = \frac{3K(1 - 2\nu)}{2(1 + \nu)} \tag{2}$$

$$G = \frac{E}{2(1 + \nu)} \tag{3}$$

Analyzing the presented equations, it can be easily observed that when the Poisson’s ratio decreases, the value of the shear modulus and consequently the shear resistance increases.

In Fig. 9, the bulk and shear moduli of isotropic solids are graphically correlated with the Poisson’s ratio. It can be observed that for stable unconstrained solids, the shear modulus must be positive [71]. This implies that the Poisson’s ratio has values between -1 and 0.5, which is the isotropic solid limit. This relationship causes that at the extreme negative values of Poisson’s ratio the shear modulus tends to infinity.

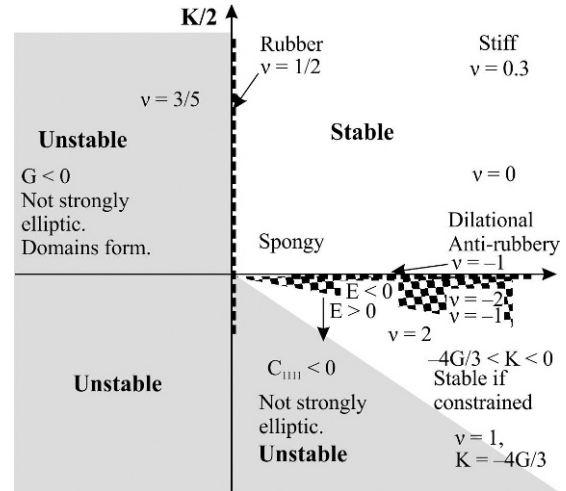


Fig. 9. Correlation of the bulk and shear moduli with the Poisson’s ratio and stability [72].

3.3. Fracture resistance

Materials that possess a negative Poisson’s ratio have a better resistance to fracture than “regular” materials [73, 74]. They also have low crack propagation [75] and more energy is necessary to expand them than in case of “regular” materials [76]. Thus, these kinds of materials have a fragile fracture.

In his work on the crack growth, Maiti demonstrated that the stress intensity factor for conventional foams (K_{IC}^*) is proportional to the normalized density and can be described by equation 4 [77]:

$$\frac{K_{IC}^*}{\sigma_f \sqrt{\pi l}} = 0.19 \left(\frac{\rho^*}{\rho_s} \right) \tag{4}$$

where σ_f is the fracture stress of the cell rib, l is the rib length, ρ^* is the foam density and ρ_s is the density of the foam based material.

Later, the work by Choi and Lakes showed that in the case of reentrant foams, Eq. 4 was not applicable and that the stress intensity factor for this kind of foams (K_{IC}^r) could be expressed by equation 5 [73]:

$$\frac{K_{IC}^r}{\sigma_f \sqrt{\pi l}} = 0.1 \frac{\sqrt{1 + \sin(\frac{\pi}{2} - \varphi)} \rho^*}{1 + \cos(2\varphi) \rho_s} \tag{5}$$

where φ is the rib angle of the reentrant cell, presented in Fig. 10.

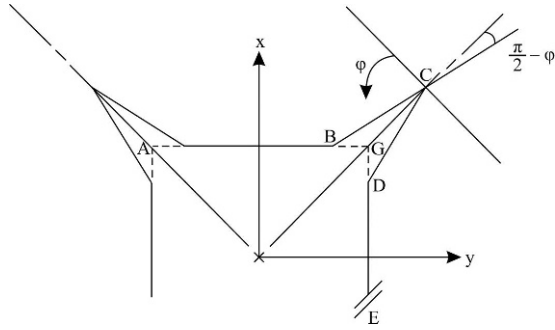


Fig. 10. Schematic cross-section view of a reentrant cell [73].

In the same work Choi observed that for the analyzed reentrant foam, the relationship between stress intensity factors could be established according to equation 6 [9]:

$$\frac{K_{IC}^r}{K_{IC}^*} = 0.53 \frac{\sqrt{1 + \sin(\frac{\pi}{2} - \phi)}}{1 + \cos(2\phi)} \quad (6)$$

Experimental results also showed that for higher values of volumetric compression, reentrant foams revealed an increased fracture toughness, as demonstrated in Fig. 11.

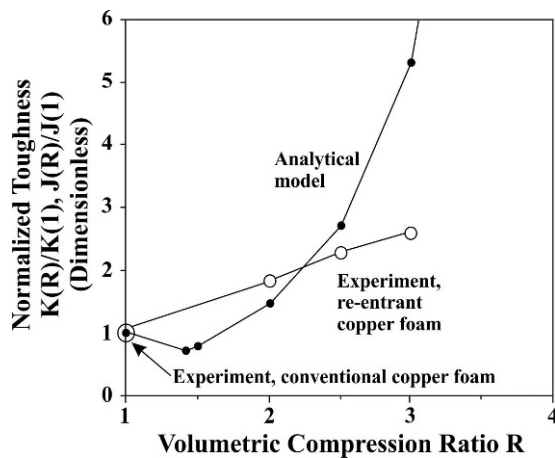


Fig. 11. Experimental normalized fracture toughness. Open symbols: conventional foam and solid symbols: reentrant foam [73].

This phenomenon can be explained by the basic definition of auxetic materials. When these materials are submitted to a tensile strength, they increase their dimensions. This dimensional growth

is verified macroscopically. However, the visualized growth is only the result of the dimensional increase of each individual auxetic cell. This way, whenever a crack is formed, the expansion of the cell will tend to close it.

3.4. Acoustic absorption

Auxetic foams have a superior capacity of acoustic absorption than conventional foams [78, 79]. The auxetic structure plays a relevant role in the attenuation of acoustic vibrations. Their performance is more relevant in frequencies under 1500 [Hz] [80].

An example of the magnitude of this effect is the study of *ultra-high-molecular weight polyethylene*. This material presents an ultrasonic attenuation value 1.5 times higher than a sintered foam and 3 times higher than a conventional foam, composed of the same base material [81].

Another example of the superior performance of auxetic materials was observed by Ruzzene while studying the attenuation of elastic waves over certain frequency bands (*stop bands*) and the directional characteristics in hexagonal honeycombs and auxetic (*bowtie*) lattices [82]. The studied periodic structures are presented in Fig. 12.

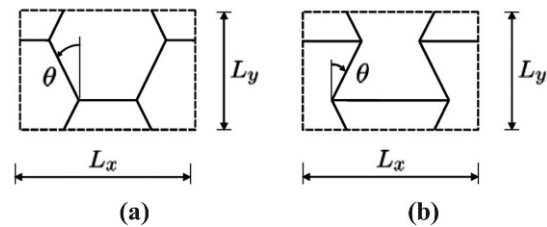


Fig. 12. Hexagonal honeycombs (a) and auxetic (bowtie) lattice (b) unit cells [82].

Numerical results demonstrated that for a rib angle $\theta = 30^\circ$ ($\theta = -30^\circ$, for auxetic cells), the auxetic structures showed a superior wave attenuation in the great majority of that structures orientation angles (ϕ) and wave frequencies (ω), as shown in Fig. 13.

3.5. Synclastic behavior

Synclastic behavior is a body's ability to deform in a shape of a dome when it is bent [83].

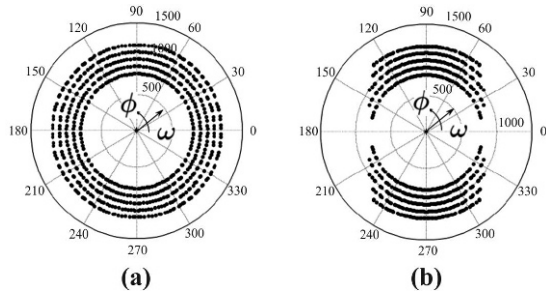


Fig. 13. Band gap representation in hexagonal honeycombs (a) and auxetic (bowtie) lattices (b) [82].

Reviewing the basic concepts of mechanics of materials, when a body is bent, it is submitted to tensile and compression stresses. Consider the concavity formed by the bending deformation. Focusing on the case of auxetic materials, there is an expansion and a contraction of the material in the exterior and the interior of the material, respectively. While bending the auxetic material, a dome is formed [84], as a result of expansion of the pulled material and the contraction of the compressed portion. This behavior is shown in Fig. 14.

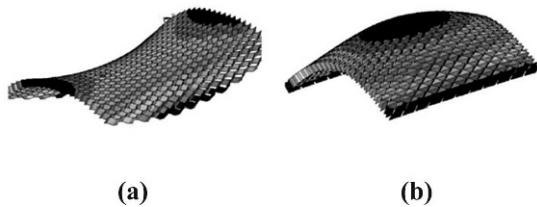


Fig. 14. Anticlastic hexagonal honeycomb (a) and synclastic auxetic reentrant structure (b) [85].

The ability to form the doubly curved shapes is useful [86], e.g. because it provides a way to fabricate this kind of complex structures without the necessity of using damaging techniques nor additional machining [87], which are normally used to obtain such shapes [61].

3.6. Variable permeability

Due to the expansion and contraction behavior, namely in auxetic foams, it can be said that these structures have variable permeability.

In auxetic foams, the variation of the structure dimensions is the reflection of the change of the dimensions of each individual cell. Consequently, it

can be seen that each cell of the structure is nothing but a pore that can be opened and closed [49] in the more convenient way. This characteristics can be observed in Fig. 15.

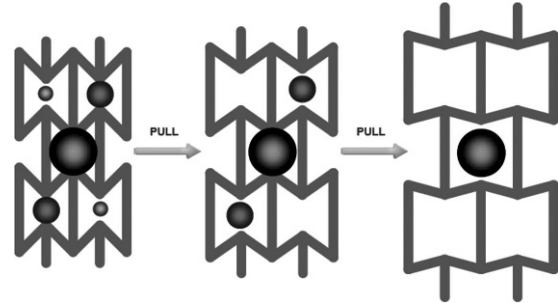


Fig. 15. Variable permeability [36].

3.7. Shape memory auxetics

Shape memory is the ability of a material subjected to a plastic or semi-plastic deformation to remember and return to its initial shape and size, when submitted to a specific thermal stimulation [88, 89]. Studies in this field show that it is possible to obtain auxetic foams that can be reverted to conventional foams several times without loss of mechanical characteristics [90, 91]. This property is extremely useful in situations that require auxetic and non-auxetic variable mechanical properties [92], where the variation of temperature is involved [93].

3.8. Other auxetic properties

The improved fracture toughness and hardness of auxetic materials suggest that these materials can have better tribological attributes than conventional materials. This can be justified by the properties that reduce the abrasive wear in auxetic materials.

This possibility was confirmed in the work by Uzun who showed that auxetic based weft knitted fabric had an increase of 15 – 35 % of abrasive wear resistance when compared to conventional polypropylene knitted fabrics [94].

Another interesting property of auxetics is the dielectric behavior in chiral honeycombs. It was suggested by Kopyt that the panels composed of hexa-chiral honeycombs may act like a homoge-

nous medium, despite their complex and heterogeneous geometry [95].

4. Applications

Considering the properties that are characteristic of these materials thanks to their negative Poisson's ratio, there were created conditions for development of new potential applications and mechanisms that otherwise would be impossible to obtain [96].

One possible application of these materials is the manufacturing of piezoelectric sensors [97]. The low bulk modulus [65] and the capacity to obtain an auxetic matrix that can follow the deformation of the piezoelectric rods [64], makes this sensors more sensible to the variation of pressure. This behavior can be observed in Fig. 16.

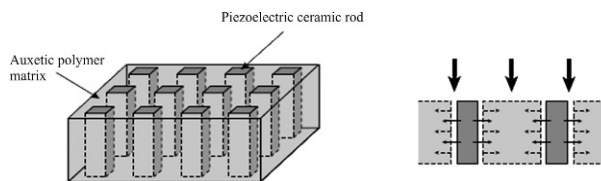


Fig. 16. Auxetic piezoelectric sensor [64].

As it was mentioned earlier, one of the most desirable properties of these materials is the variable permeability. This property can be used in manufacturing of intelligent filters, which can be designed with different sizes and particular geometries to control the passage pressure while filtering [98].

An interesting field of application for these materials is biomedical engineering. The evolution of these materials will allow the manufacturing of blood vessels that expand their walls when the blood is pumped [99] or the development of new surgical tools and mechanisms. Consider, for example Fig. 17, where a possible blood vessel dilator is represented.

Taking into account that the most obvious characteristics of these materials is their expansion when submitted to a tensile load, there can be found common applications that use this as an advantage. One studied application is the use of auxetic fasteners. These fasteners contract when inserted and

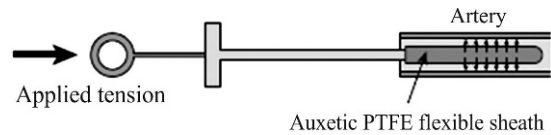


Fig. 17. Blood vessel dilator [64].

expand on an attempt of removal. In this way, a bigger force is required to remove them [100].

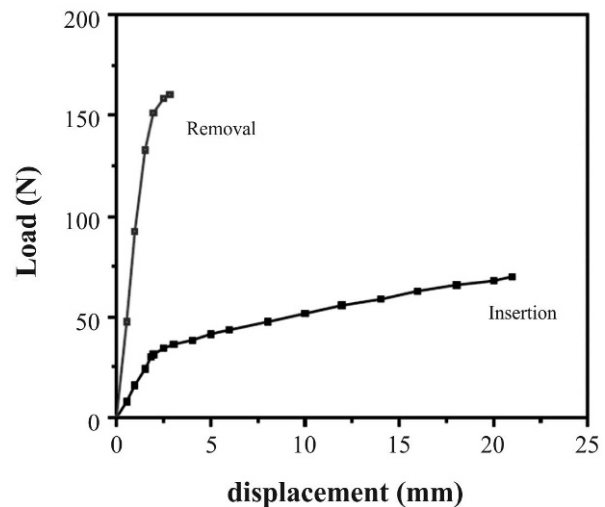


Fig. 18. Behavior of auxetic fastener [100].

It can be observed in Fig. 18 that the expansion originating during the removal required bigger load than for the fastener insertion.

Another example of the application of auxetic materials is the manufacturing of a chiral-based honeycomb deployable antenna for deep-space missions, using the shape-memory properties [89]. This antenna is folded while transported, as there is a very limited area for rocket launchers. Once in space, the shape memory structure uses the thermal energy of the sun to unfold to its original size, as shown in Fig. 19.

Auxetic textile structures are also a fast growing field. Their development should lead to the fabrication of materials with improved energy absorption, high volume change, wear resistance and drapability [101]. Some of these characteristics can be used in the aerospace, automotive and military sector [102].

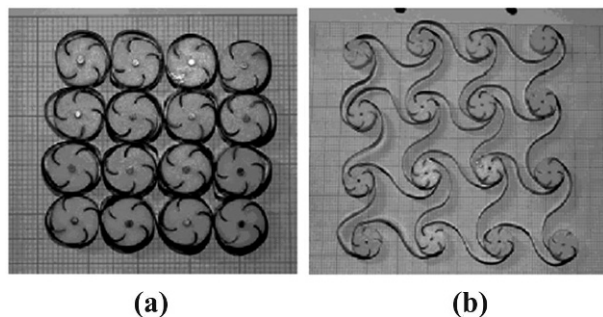


Fig. 19. Folded (a) and unfolded (b) auxetic cellular antenna [89].

The production of this kind of textiles can be executed by two basic methods. The first one is the use of auxetic based fibers directly in the knitting and weaving of the textiles [103]. The other one is the production of auxetic textiles using conventional fibers weaved or knitted into a structure that is auxetic by itself [101, 104].

The use of auxetic textiles can already be found commercially, for example in applications that use *GoreTex* and *polytetrafluorethylene* [66].

One of the most promising fields is the development of auxetic materials at a nanoscale, for example in the applications with carbon nanotubes. These kinds of materials are basically made of the molecules composed of a monolayer of carbon atoms arranged in a cylindrical lattice [105]. One of the applications of this nanostructured auxetics can be the molecular variable permeability filters [106].

The simulations that represent these nanostructures suggest that under certain geometric and force conditions, when the stretching deformation in the cell walls dominates, it is possible to obtain negative values of Poisson's ratio [105].

5. Conclusions

As a relatively new class of materials, auxetics are generating a progressive interest in the scientific community. Their counter-intuitive behavior, allowed by the deformation mechanism of the internal structure, gives new perspectives in terms of possible applications of these materials.

Although this kind of behavior is not rare in anisotropic materials (partial auxetics), there are

real advantages in the continuous development of isotropic materials with negative Poisson's ratio (auxetics). This objective is gradually achieved by the constant study of the theoretical structures, like reentrant, rigid and semi-rigid as well as chiral models.

The unique combination of mechanical characteristics, like their superior resistance to indentation, shear, fracture and wear, make them very desirable at a structural level. On the other hand, the advantages generated by their variable permeability, acoustic absorption, synclastic behavior and their evolution in the shape memory field make them very promising as technological materials.

The continuous study of these materials at a molecular level, for example in a nano scale may provide the means to obtain new homogenous auxetic materials that combine the advantages of both regular and auxetic materials.

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