Statistical Model of College Students' Mental Health Based on the Law of Large Numbers

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Abstract

This paper constructs a mental health probability space based on the law of large numbers. First, this paper defines and predicts the transition of personal emotion and the transmission of emotion. This study treats emotion as two random stages. At the same time, according to the initial parameters of each different stage model, a psychological model with individual characteristics based on the Markov chain is established. It can predict the outcome of emotional activities emotionally. Finally, numerical simulation is carried out with Matlab. The results show that the prediction method can well reflect the emotional state transmission. It can be used in psychological prediction simulations of individuals. The mental health statistical model based on the law of large numbers established in this paper opens up a new way for individual psychological counseling and mental health training.

Keywords: Psychological prediction model; Law of large numbers; Central limit theorem; Hidden Markov model

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1 Introduction

Although emotional information processing is a long-standing concern in cognitive science and artificial intelligence, it has not been well answered. In addition to standard logic, human intelligence should also have emotional processing. Artificial psychology is a behavior that completes the analysis of human thought and behavior through computer technology. The artificial psychology model aims to simulate human thought and behavior through computer simulation comprehensively, so it has multiple functions such as intelligence, emotion, will and character. The emotion pattern is the emotion recognition pattern in emotion computation [1]. One is a mental model based on emotion analysis, and the other is an image recognition model based on emotion. The main focus of this thesis is on emotional patterns. This is the driving force of emotions. This emotional engine uses sensed tangible conscious signals as its input and uses it to simulate emotional responses. This generates an output of emotion. Some scholars have constructed a four-level dynamic interaction system from the aspects of neuroscience, philosophy, and psychology. Some scholars use neural network technology to simulate the subject's emotion simulation in virtual reality. Due to the superiority of the law of large numbers, we chose the Markov stochastic model in the law of large numbers to model emotions. This paper firstly establishes a "sentiment analysis model" based on the "law of large numbers." In this paper, "feeling" is regarded as a "signal" reflecting the natural psychological state. At the same time, this paper regards "feeling" as a "Markov" "perception."

Furthermore, this paper uses the signal pattern of the law of large numbers to describe emotional processing. The study found that the model achieved the expected results [2]. The psychological law of the model of the large number proposed in this paper can make us better understand the source of the signal - the change of emotion so that we can simulate the source and generation of the signal.

2 Several forms of the law of large numbers

There are many forms of the law of large numbers in academia today. Here is a brief description of some of the most common laws of the Law of Large Numbers:

Theorem 1. (Chebyshev's Law of Large Numbers)

Assume that the arbitrary variables $\delta_1, \delta_2, \ldots, \delta_n, \ldots$ are uncorrelated with each other. Its variances are in $\omega_1^2, \omega_2^2, \ldots, \omega_n^2, \ldots$ order. There is also a positive number $h$ which is a standard number for which all $i = 1, 2, \ldots, L$ have $\omega_i^2 \leq h, (h > 0)$. So any ordinary value $\mu$ has

$$\lim_{n \to \infty} P\left(\left|\frac{1}{n} \sum_{i=1}^{n} \delta_i - \frac{1}{n} \sum_{i=1}^{n} W(\delta_i)\right| < \mu\right) = 1$$

(1)

Proof: Chebyshev's inequality applies as follows:

$$Q\left(\left|\frac{1}{n} \sum_{i=1}^{n} \delta_i - \frac{1}{n} \sum_{i=1}^{n} W(\delta_i)\right| \geq \varepsilon\right) \leq \frac{\left(\frac{1}{n} \sum_{i=1}^{n} \delta_i\right)}{\varepsilon^2} = \frac{1}{n^2} \frac{\sum_{i=1}^{n} \delta_i}{\varepsilon^2} = \frac{\phi\left(\frac{\sum_{i=1}^{n} \delta_i}{\varepsilon^2}\right)}{n^2} \leq \frac{1}{n^2 \varepsilon^2}$$

(2)

Since $\delta_1, \delta_2, \ldots, \delta_n, \ldots$ is a set of random variables that are independent of each other. Its variance is bounded. Therefore, we can draw the following conclusions:
\[ \phi \left( \sum_{i=1}^{n} \delta_i \right) = \sum_{i=1}^{n} \phi(\delta_i) \leq nh \]  

(3)

So there will be an

\[ Q \left( \frac{1}{n} \sum_{i=1}^{n} \delta_i - \frac{1}{n} \sum_{i=1}^{n} W(\delta_i) \right) \geq \varepsilon \leq \frac{h}{n^2} \rightarrow 0, n \rightarrow \infty. \]

This rule can be proved.

**Theorem 2.** (Bernoulli's Law of Large Numbers)

Suppose \( n_G \) is the number of occurrences of \( G \) in \( n \) times Bernoulli experiments. \( p \) is the probability of occurrence of \( G \) in each experiment. For any positive \( \mu \), it has

\[ \lim_{n \to \infty} Q \left( \left| \frac{n_G}{n} - p \right| < \mu \right) = 1. \]

This rule states that the number of occurrences of \( G \) in a Bernoulli experiment with \( n \) is almost the same as the probability of occurrence of \( G \) in each experiment. The theory is an exact representation [3]. In practical applications, when the number of trials is large, the frequency of the event can be used to replace the probability of the event.

**Theorem 3.** (Poisson's Law of Large Numbers)

Suppose that the probability of a class \( G \) accident in an experiment \( i \) is \( p_i, i = 1, 2, \ldots, n \). \( n_G \) is a Category \( G \) accident in Phase \( n \) clinical study with

\[ \lim_{n \to \infty} Q \left( \left| \frac{n_G}{n} - \sum_{i=1}^{n} p_i \right| < \mu \right) = 1. \]

Poisson's law of large numbers shows that its frequency remains stable under different conditions in random experiments. The number of occurrences of event \( B \) in \( n \) independent experiments gradually becomes stable as \( n \) increases infinitely. It is close to the mathematical mean of the odds of \( G \) in each experiment.

### 3 The spatial and temporal forms of the three emotions

This article divides emotions into three different emotional spaces: happy, calm, and unhappy. They are consistent with the emotion quantity \( M \) in the hidden Markov pattern [4]. The self-evaluation and other-evaluated emotional indicators are regarded as independent emotional indicators corresponding to the individuals observed in the hidden Markov model. A person's emotional scale in this "happy" state is 1 to 7. 1 is mildly happy, and 7 hours is manic. Use 0 to represent the emotional index of calmness. In the state space of "unhappy," the emotional index is defined as -1 ~ -7, where -1 represents mild unhappiness, and -7 represents depression.

This paper uses the traditional discrete law of significant numbers method to predict the psychological. Emotional indicators of self-evaluation and others' evaluation may appear in different implicit emotional states. Hidden Markov Theory includes two aspects. That is, the Markov chain represented by \( \pi, A \). It is evident that \( \pi \) and \( A \) will lead to different shapes of the Markov chain. Due to the
differences in genes and the environment of growth in the individual's psychological expectation model, the form of the stable emotional state within each person is also different [5]. The model can reach any condition at any point in time. Its Markov chain is a state traversal.

4 Approaches to Establishment of Psychological Forecasting Models

In this paper, the Baum-Welch method of the law of large numbers is used to estimate the possibility of emotion transmission and the probability vector of various outputs of emotion.

\[
W = -\varphi \sum_{i=1}^{n} q_{\alpha} \log q_{\alpha}
\]  

(4)

\( W \) is the entropy of emotion, which is the number of emotions. \( B \) represents the probability of occurrence of the \( \alpha \) emotional condition. \( \varphi \) is a constant value for the log base and element selection. The probability of occurrence of each emotion in the emotion space is the same,  

\[
q_{\alpha} = \frac{1}{n}, \alpha \in (1, 2, \cdots, n^n).\]  

This will maximize emotional entropy

\[
W_{\text{max}} = -\varphi \log q_{\alpha} = \varphi m \log n
\]  

(5)

\( W_{\text{max}} \) represents the emotional maximum of the organisms investigated in this paper. \( W_{\text{max}} \) is the upper limit, and the closer to this value, the stronger the emotion. This emotional entropy can reflect the overall expression of the emotion constructed in this paper [6]. And emotional stability in a specific time can be expressed by emotional entropy.

\[
w_{ij} = -\varphi \sum_{j=1}^{n} q_{ij} \log q_{ij}
\]  

(6)

\( w_{ij} \) is the sentiment entropy value of stage \( i \). \( q_{ij} \) is the probability of the emotional state in the \( i \) to the \( j \) emotional state. \( \alpha \) is a constant value for logarithmic base and unit selection. Therefore, the following inferences are made about emotional states and their associated problems.

1) The probability of each emotional condition at any point in time sums to 1. This is also the sum of the energies assigned to each emotional situation and the total energies at any time.

2) The sum of the emotional intensity of all emotional conditions at any time is 0. That is to say, different emotions are mutual [7]. The higher the intensity of one emotion, the lower the intensity of other emotions.

3) The energy change of emotion should be continuous and not mutated. The energy distribution \( e_{\alpha}^{i} (i = 1, 2, 3) \) at time \( k \) can be assumed to be related only to the energy distribution \( e_{\alpha}^{i-1} (i = 1, 2, 3) \) at the last time. It has no relation to the energy distribution \( e_{\alpha}^{i-2}, e_{\alpha}^{i-3}, \cdots, e_{\alpha}^{0} (i = 1, 2, 3) \) at previous times. It can also be considered that the probability distribution map \( q_{\alpha}^{k} (i = 1, 2, 3) \) of emotional status at time \( k \) is only related to \( q_{\alpha}^{k-1} (i = 1, 2, 3) \) at the previous time. It is not affected by last time. This is precisely the non-lag property of
Markov processes. For this reason, this paper can use the Markov chain to express the transmission of emotion.

5 Application of Psychological Prediction Models

In this paper, the psychological prediction method of the Markov stochastic model can well describe a person's emotional state. The individual has an inherent, usually stable state of everyday life that does not cause substantial emotional changes [8]. This stable state is the average emotion of the individual. It is in such emotions that individuals usually connect with the outside world. This paper establishes a psychological expectation model. This pattern can describe a person's emotional pattern when there is no significant change in life status during a period.

For example, let Student A start self-evaluation from the mood after getting up in the morning. Students are assessed by three different emotional indicators [9]. Also, let Student B keep in touch with Student A. Student B asks Student A to report on himself every 30 minutes. Student B records Student A's mood in the observation sheet as Student A. In this experiment, Student A conducts a self-evaluation every half hour from 7:00 in the morning. By the time they went to bed at night, the students had completed 33 self-assessments. Classmate B also made his evaluation 33 times. This article uses student A as an example to draw a table. In Figures 1 and 2, the horizontal axis represents the measured quantity, and the vertical axis represents the sentiment index.

![Figure 1. Self-Assessment Sequence](image-url)
After analyzing Student A's self-evaluation, we tested the Adolescent Life Events Inventory (ASLEC). The study found that he was not under severe stress for over three months. This shows that Student A's emotion is an inner emotion [10]. This paper uses the proposed psychological prediction model to calculate the transfer moment A of individual emotional state and the probability moment B of the emotional intensity distribution. This is also known as maximum likelihood estimation.

$$\begin{bmatrix} 0.50 & 0.20 & 0.30 \\ 0.20 & 0.80 & 0.00 \\ 0.25 & 0.08 & 0.66 \end{bmatrix}$$

$$\begin{bmatrix} 0.00 & 0.00 & 0.00 & 0.00 & 0.10 & 0.20 & 0.40 & 0.30 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.63 & 0.36 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.25 & 0.25 & 0.33 & 0.08 & 0.08 & 0.00 & 0.00 \end{bmatrix}$$

This paper predicts a person's emotional state from two perspectives, A and B (Figures 3 and 4). The graph's horizontal axis represents the measured quantity, and the vertical axis represents the sentiment index.
The chart shows that Student A's emotional status in the follow-up observation of the following days is consistent with the trend of emotional changes expected by this model [11]. The results show that the established mathematical model conforms to reality.

For example, if a person suffers a significant life crisis for more than a month, his mood will change. If this is the case for Student C. Then, this paper can get that the $A_i$ matrix before the life activity is $Q$, that is,
The A matrix after life events is $A_2$, i.e.

$$A_2 = \begin{bmatrix} 0.60 & 0.20 & 0.20 \\ 0.80 & 0.10 & 0.10 \\ 0.90 & 0.05 & 0.05 \end{bmatrix}$$

This paper analyzes the transfer matrix $A$ of its inner emotional state. In this paper, through Matlab simulation, Student C's inner personality emotion development curve is obtained before and after life activities. From two different emotional sequences, when a person is attacked, his emotional transmission pattern will change [12]. The likelihood of his negative emotions turning into positive one's decreases, and his chances for negative emotions turning into negative ones increase. This changes the balance in his emotional development curve. The emotional development curve $(x, y, z)$ represents the possibility of a person's unpleasant, peaceful and happy moods. It can be seen from the numerical value that the emotional convergence value has changed from $(0.05, 0.3333, 0.6167)$ to $(0.7667, 0.1167, 0.1167)$. These expectations allow teachers to intervene properly in student C appropriately. Teachers can provide corresponding help according to the change of their dynamic transmission mode to eliminate the negative emotional state in time. This predictive effect provides scientific, objective, and specific information about the individual's emotions. This predictive effect provides scientific, objective, and specific information about the individual's emotions. The basic principles and simulation methods of affective computing described in this paper are consistent with the relationship between human emotions. This lays a solid foundation for further exploration of the development of psychology. This also provides a new idea for a more reasonable emotion prediction calculation model.

6 Conclusion

This paper uses Hidden Markov Models and the Law of Large Numbers to apply some experiential concepts to predict personal emotions. This paper considers emotional information as a sequence of observations generated by emotional processes. The model can personally evaluate the emotional information that is most likely to be generated in the future by predicting it. Finally, this paper uses Matlab software to model it. This paper is programmed and implemented by the Baum-Welch model. The model can lay a foundation for theoretical research on the parameter selection problem of affective patterns. This paper uses the theory of emotional entropy and the law of large numbers to analyze the impact of personal emotional development. The results show that the method is consistent with an individual's emotional development process. This model is a better prediction method.

References


