Design of a MAGLEV System with PID Based Fuzzy Control Using CS Algorithm

B. Ataşlar-Ayyıldız, O. Karahan

Kocaeli University, Dept. of Electronics and Communication Engineering, Kocaeli, Turkey
E-mails: banu.ayyildiz@kocaeli.edu.tr  oguzhan.karahan@kocaeli.edu.tr

Abstract: The main aim of this study consists of proposing a simple but effective and robust approach for PID type fuzzy controller (Fuzzy-PID) in order to improve the dynamics and stability of a magnetic ball levitation system. The design parameters of the proposed controller are optimally determined based on Cuckoo Search (CS) algorithm. During the optimization, a time domain objective function is used for minimizing the values of common step response characteristics for the optimal selection of the controller parameters. Robustness tests are performed to evaluate the performance of the proposed controller through extensive simulations under load disturbance, parametric variation and changes in references. Moreover, to show the advantage and compare the performance of the proposed controller, the PID and Fractional Order PID (FOPID) controllers tuned with CS are designed. The simulation results and comparisons with the CS based PID and FOPID controllers demonstrate that the CS based Fuzzy-PID controller has superior performance depending on small overshoot, short settling time, fast rise time and minimum steady state error. Compared with the PID and FOPID controller tuned with CS, the simulation results show that the proposed Fuzzy-PID controller tuned with CS outperforms in terms of the accuracy, robustness and the least control effort.

Keywords: Cuckoo Search algorithm, MAGLEV, Fuzzy Logic Control, Fractional order PID, PID.

1. Introduction

A MAGnetic LEVitation (MAGLEV) device works based on the principle that a steel ball may be levitated in the air by the force generated by an electromagnet. Therefore, the steel ball is lifted in the air gap and its position can be controlled in a continuous fashion. On the other hand, the system is eminently nonlinear, and also unstable due to the same reason. These challenges have appealed increased number researchers in the past few years, and, consequently, MAGLEV system (Maglev system) has become widely known in several industries as well. As the outcome, MAGLEV
systems have been progressively utilized in a variety of industrial applications such as microelectromechanical systems [1], high-speed magnetic levitation trains [2], bearingless motors [3], vibration isolation [4], wind tunnel [5] and magnetic bearing [6].

Conventional PID controllers are widely used in many linear and nonlinear industrial systems and applications as they are easily applied, and also they can provide satisfactory performance in a variety of industrial control problems [7]. On the other hand, if external disturbances and parameter uncertainties exist, a PID controller may not provide desired control accuracy in robustness and performance perspective. Hence, for attaining the requisite performance with a PID controller, numerous methods have been established and offered. Podlubny [8] proposed a generalized form of a conventional PID controller, which is denominated as the Fractional Order PID (FOPID), based on extending the order of integration and differentiation. In the recent past, researchers developed and offered FOPID controllers for various applications such as automatic voltage regulator [9], wind energy systems [10], motion and tracking control [11], designing aerospace control systems [12], varying time delay processes [13].

For nonlinear and complex dynamical systems, there have been recent researches on nonlinear controllers, mostly by using fuzzy logic principles [14]. As a consequence of these studies, it has been acknowledged that the Fuzzy Logic Controllers (FLCs) outperform traditional controllers, such as conventional PID controllers, by providing more robustness in such applications that contains complex, nonlinear and coupled systems, uncertain systems and systems where exact dynamics is not required or available. From the group of different FLCs, such as Fuzzy-PD, which has extensive usage, Fuzzy-PI, and Fuzzy-PID, in numerous systems, fuzzy-based PID controllers have recently been most preferred solution to overcome problems in controlling complex nonlinear dynamical systems. The principle of PID type fuzzy controller used in this study incorporates Fuzzy-PD and Fuzzy-PI controllers with the gains as the input scaling factors and the gains as the output scaling factors as described by [15, 16].

In a Fuzzy-PID controller, plausible tuning parameters could be the selection of Membership Functions (MFs), input variables, scaling factors, rule base, fuzzification, inference and defuzzification techniques. For tuning the parameters of the Fuzzy-PID controllers different tuning methods have been developed and available in the literature such as Particle Swarm Optimization (PSO) [17], Genetic Algorithm (GA) [18], Firefly Algorithm (FA) and Teaching Learning Based Optimization [19].

Maglev systems exhibit outstanding nonlinear and uncertain behaviour. In control theory point of view, this brings several challenges to the design of a controller. To overcome those difficulties in designing a controller for position control of the levitated object in a MAGLEV system, variety of techniques have been developed by different researchers. Naturally, by optimizing their parameters utilizing various approaches, PID controllers has been used in controlling MAGLEV systems as well [20-27]. Several researchers have developed different FOPID controllers to obtain stable levitating and reinforced trajectory tracking control of
Maglev system [28-35]. Fuzzy control methods have also been employed in some studies on Maglev systems [36, 37].

In this study, implementations of Fuzzy-PID, FOPID and PID controllers tuned with Cuckoo Search (CS) algorithm is one by one exercised in the Maglev system to investigate the superior results with regard to tracking and disturbance rejection. CS based Fuzzy-PID controllers’ performance is analysed based on certain indices including step response and results are compared to CS based PID and FOPID controllers’ results. Moreover, comparison of the robustness tests are considered to demonstrate that the Fuzzy-PID controller outperforms controllers with other tuning methods.

This study is arranged as follows: in Section 2, a MAGLEV system’s dynamic model is provided. Section 3 contains an outline of the CS algorithm. The Fuzzy-PID controller is described in Section 4. Other controller’s design is also included in this section. Section 5 contains simulation results including a review of results. Then, in Section 6, which is final section, a conclusion is presented.

2. MAGLEV system model

Essentially, MAGLEV system stems from the fact that a ferromagnetic mass may resist gravity through a magnetic field created by a coil through which electrical current flows. In other words, a Maglev system is basically an object, ferromagnetic in nature, that overcomes the gravity in a magnetic field, which can be adjusted by the voltage applied to the coil. As shown in Fig. 1, two forces are applied to the ferromagnetic object, i.e., ball: (1) its weight due to the gravity, and (2) the electromagnetic force generated by the magnetic field of the coil, through which current flows when a voltage is applied.

![Block-diagram of the Maglev system](image)

Fig. 1. Block-diagram of the Maglev system

The relationship between the ball displacement, $x$, and the current which passes through the coil of the electromagnet, $i$, in a Maglev system is given by [25] as

$$m\ddot{x} = mg - k\frac{i^2}{x^2}.$$
where $m$ is the mass of the levitated ferromagnetic ball, $g$ is the gravitational constant, and $k$ is an constant of electromechanical conversion. This nonlinear equation that models the Maglev system can be linearized around the equilibrium point $x_0$ and $i_0$ as

$$\Delta X(s) = \frac{-k_i}{s^2-k_x},$$

where, $k_i = \frac{2g}{i_0}$, $k_x = \frac{2g}{x_0}$, and $\Delta x$ and $\Delta i$ are the small derivations from the equilibrium point $x_0$ and $i_0$, respectively. Because the coil current $i$ is proportional to the control voltage $u$, i.e., $i = k_1 u$, and the sensor output $x_v$ is proportional to the position of the ball $x$, i.e., $x_v = k_2 x$, the transfer function from $\Delta u$ to $\Delta x_v$ can be obtained as

$$\frac{\Delta x_v(s)}{\Delta u(s)} = \frac{-k_1 k_2}{s^2-k_x},$$

where $k_1$ is the control voltage to coil current gain and $k_2$ is the sensor gain. Ultimately, by substituting system parameter values given in Table 1, which has been taken from [25], into Equation (3):

$$G(s) = \frac{\Delta x_v(s)}{\Delta u(s)} = \frac{-3518.85}{s^2-2180}.$$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of the ball (kg)</td>
<td>$m$</td>
<td>0.02</td>
</tr>
<tr>
<td>Acceleration due to gravity (m/s²)</td>
<td>$g$</td>
<td>9.81</td>
</tr>
<tr>
<td>Equilibrium value of current (A)</td>
<td>$i_0$</td>
<td>0.8</td>
</tr>
<tr>
<td>Equilibrium value of position (m)</td>
<td>$x_0$</td>
<td>0.009</td>
</tr>
<tr>
<td>Control voltage to coil current gain (A/V)</td>
<td>$k_1$</td>
<td>1.05</td>
</tr>
<tr>
<td>Sensor gain (V/m)</td>
<td>$k_2$</td>
<td>143.48</td>
</tr>
</tbody>
</table>

3. Cuckoo search optimization algorithm

Cuckoo Search (CS) optimization algorithm is known as the most effective swarm-intelligence based algorithm, in which the global random walk is carried out by utilizing Lévy flights law to generate new nests as

$$X_i(n+1) = X_i(n) + \alpha \text{levy}(\lambda),$$

where: $n, n = 1, ..., N$, indicates the current iterations in which $N$ denotes the predetermined maximum iteration number; $\alpha > 0$ is the step size, which is determined according to the scale of the problem of interest; levy($\lambda$) represents Lévy flight process which is basically a random walk derived from the Lévy distribution with an infinite variance and infinite mean [38], and

$$\text{levy}(\lambda) = t^{-\lambda}, \ (1 < \lambda \leq 3),$$

where $t$ is the current iteration. It is also possible to extend the algorithm to more complex cases where each nest may contain multiple eggs (a set of solutions) [38]. In such a case, the new nest is randomly generated by applying the equation
\( X_{\text{new}}^{i} = \begin{cases} X_{i} + \text{stepsize} \cdot \text{randn} & \text{if } \text{randn}_{i} > p_{a}, \\ X_{i} & \text{else} \end{cases} \)

where:

\[ \text{stepsize} = 0.01 \cdot \left( \frac{\sigma(\beta) \cdot \text{randn}}{\text{randn}} \right)^{\frac{1}{\beta}} \cdot (X_{i} - X_{\text{best}}); \]

\( \text{randn} \) is a random value in \([0, 1] \); \( \beta \) is a constant between \( 1 \leq \beta \leq 3 \); the standard deviation function \( \sigma(\beta) \) is

\[ \sigma(\beta) = \left( \frac{\Gamma(1+\beta) \cdot \sin(\pi \beta/2)}{\Gamma\left(\frac{1+\beta}{2}\right) \cdot 2^{(\beta-1)/2}} \right)^{\frac{1}{\beta}}. \]

In accordance with above, Cuckoo search optimization algorithm can be outlined in the following steps:

**Step 1.** Introduce \( X_{i} \) as a random population of \( n \) host nests.

**Step 2.** Generate a new solution \( X_{\text{new}}^{i} \) by using Lévy flights.

**Step 3.** Calculate its objective function \( J(X_{\text{new}}^{i}) \).

**Step 4.** Select a nest randomly among the host nests say \( X_{j} \) and calculate its objective function value as \( J(X_{j}) \).

**Step 5.** If \( J(X_{\text{new}}^{i}) < J(X_{j}) \), then replace \( X_{j} \) by new solution \( X_{\text{new}}^{i} \), else let \( X_{j} \) be the new solution.

**Step 6.** By using Lévy flights manner, leave a fraction of \( P_{a} \) of the worst nest by building new ones at new locations.

**Step 7.** Keep the current optimum nest.

**Step 8.** Go to Step 2 if max iteration number is not reached.

**Step 9.** Find the optimum solution: \( X_{\text{best}}^{i}, i = 1, \ldots, n \).

4. Controller design

4.1. Fuzzy-PID controller

The structure of the Fuzzy-PID controller assumed in this study is shown in Fig. 2. This structure is a combination of Fuzzy-PD and Fuzzy-PI controllers with \( Se \) and \( Sce \) as input scaling factors and \( Su \) and \( Si \). Both of the FLC input and output variables are represented with seven MFs as shown in Fig. 3. For both inputs and output of FLC, except NB and PB, Gaussian membership function is used considering its prominent benefits such as smooth functions, non-zero at all points, and it also provides the actual information at all points. NB and PB are chosen as Z-shape and S-shape membership functions, respectively. The range of MFs is \([-1, 1]\) for both inputs and outputs.

The fuzzy rule table is shown in Table 2, and also the fuzzy control surface is presented in Fig. 4.
Fig. 2. Structure of Fuzzy-PID controller

Fig. 3. Membership functions of input and output variables

Table 2. Table of Fuzzy Rules

<table>
<thead>
<tr>
<th>$\epsilon(t)$</th>
<th>NB</th>
<th>NM</th>
<th>NS</th>
<th>ZE</th>
<th>PS</th>
<th>PM</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NM</td>
<td>NS</td>
</tr>
<tr>
<td>NM</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NM</td>
<td>NS</td>
<td>ZE</td>
<td>PS</td>
</tr>
<tr>
<td>NS</td>
<td>NB</td>
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<td>NM</td>
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<td>ZE</td>
<td>PS</td>
<td>PM</td>
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<td>PS</td>
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<td>PB</td>
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<tr>
<td>PB</td>
<td>ZE</td>
<td>PS</td>
<td>PM</td>
<td>PB</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 4. Membership Functions of input and output variables
4.2. FOPID controller

A FOPID controller is depicted by five parameters. In comparison to the conventional PID controllers, FOPID controllers have two more parameters in which the orders of the derivative part $\mu$ and integral part $\lambda$ are non-integer. These additional parameters bring more flexibility to the design of controller and also may lead to obtain an enhanced dynamic performance. The FOPID is defined by a transfer function as below:

$$C_{\text{FOPID}}(s) = K_p + K_i \frac{1}{s^\lambda} + K_d s^\mu.$$  \hfill (10)

4.3. Traditional PID controller

A traditional PID controller has three control elements, which are proportion, differentiation, and integration of the error signal, and it is defined by a transfer function given as

$$C_{\text{PID}}(s) = K_p + K_i \frac{1}{s} + K_d s.$$  \hfill (11)

4.4. Objective function of the optimization algorithm

The objective function to be used during the optimization is selected considering performance criteria which are overshoot ($M_p$), rise time ($t_r$), settling time ($t_s$), and steady state error ($E_{ss}$), and is defined by an equation as follows:

$$J = w_{M_p} M_p + w_{t_r} t_r + w_{t_s} t_s + w_{E_{ss}} E_{ss}.$$  \hfill (12)

In this study, numerous trials have been carried out for the optimization process by applying different values of the weighting factors ($w_{M_p}$, $w_{t_r}$, $w_{t_s}$, $w_{E_{ss}}$). Eventually, values of the weights decided to be used are $w_{M_p} = 0.9$, $w_{t_r} = 100$, $w_{t_s} = 0.9$, $w_{E_{ss}} = 1000$. These values for weights are determined from results of these trials to achieve enhanced system response.

5. Simulation results

Parameters used for tuning each of three controllers, PID, FOPID and Fuzzy-PID, simulation parameters and also input parameters of CS algorithm are given in Table 3.

The optimized scaling parameters of all the controllers applied to the Maglev system and best objective function values are reported in Table 4, according to the minimum value of each objective function among 10 runs.

A step reference trajectory is applied to the Maglev system in order to monitor both steady state and transient performance of each optimized controller. Fig. 5 shows all three designed controllers’ tracking performance in the same graph, and values of all performance criteria for each controller is given in Table 5.
Table 3. Parameters used for tuning the controllers

<table>
<thead>
<tr>
<th>PID and FOPID Controllers</th>
<th>Gains Range:</th>
<th>−5 ≤ 𝐾_𝑝 ≤ −1, −7.5 ≤ 𝐾_𝑖 ≤ −0.1, −0.2 ≤ 𝐾_𝑑 ≤ 0</th>
<th>0 &lt; 𝜆, μ ≤ 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Parameters of Approximation</td>
<td>𝜔_𝑙 = 0.001, 𝜔_ℎ = 1000, N=5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**FUZZY-PID Controller**

| Gains Range: | 0 ≤ 𝑆_𝑒, 𝑆_𝑐 ≤ 2 | −100 ≤ 𝑆_𝑖, 𝑆_𝑢 ≤ 0 |

<table>
<thead>
<tr>
<th>Simulation Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Iterations</td>
</tr>
<tr>
<td>Number of trials</td>
</tr>
<tr>
<td>Simulation Time</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input Parameters of CS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Nests</td>
</tr>
<tr>
<td>Abandon Probability</td>
</tr>
<tr>
<td>Constant in (6)</td>
</tr>
</tbody>
</table>

Table 4. Optimized parameters and performance index values of the PID, FOPID and Fuzzy-PID controllers

<table>
<thead>
<tr>
<th>FOPID and PID Controller Parameters</th>
<th>Objective (J)</th>
<th>𝐾_𝑝</th>
<th>𝐾_𝑖</th>
<th>𝐾_𝑑</th>
<th>μ</th>
<th>λ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12.5453</td>
<td>−4.9089</td>
<td>−7.0825</td>
<td>−0.1668</td>
<td>1.0162</td>
<td>1.0051</td>
</tr>
<tr>
<td></td>
<td>15.4352</td>
<td>−4.8558</td>
<td>−7.0125</td>
<td>−0.0965</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fuzzy-PID Controller Parameters</th>
<th>Objective (J)</th>
<th>𝑆_𝑒</th>
<th>𝑆_𝑐</th>
<th>𝑆_𝑖</th>
<th>𝑆_𝑢</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.2502</td>
<td>1.2569</td>
<td>0.0046</td>
<td>−98.2561</td>
<td>−95.7456</td>
</tr>
</tbody>
</table>

Fig. 5. Step responses of Maglev system with the tuned controllers using CS
Table 5. Results of Response Characteristics Obtained from PID, FOPID and Fuzzy-PID Controllers based on CS Algorithm

<table>
<thead>
<tr>
<th>Controller</th>
<th>$M_p%$</th>
<th>$t_r$, 0.1 → 0.9</th>
<th>$t_s \pm 5%$</th>
<th>$E_{ss}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID</td>
<td>15.8937</td>
<td>0.0050</td>
<td>0.6690</td>
<td>0.000029</td>
</tr>
<tr>
<td>FOPID</td>
<td>13.0045</td>
<td><strong>0.0020</strong></td>
<td>0.6790</td>
<td>0.000030</td>
</tr>
<tr>
<td>Fuzzy-PID</td>
<td><strong>0.4599</strong></td>
<td>0.0070</td>
<td><strong>0.0090</strong></td>
<td><strong>0.000027</strong></td>
</tr>
</tbody>
</table>

Considering the graph in Fig. 5 and values given in Table 5, it can evidently be concluded that the Maglev system’s performance with CS-tuned Fuzzy PID controller is superior to the performance achieved by CS-tuned conventional PID and FOPID controllers. Similarly, achieved values of each performance criteria for CS-tuned Fuzzy PID controller is better than the other two controllers.

In the next step, as shown in Fig. 6, in order to test robustness of each controller with different tuning, a square wave with −1.5V mean value and 0.5V magnitude changing at 10s intervals has been applied to the Maglev system.

![Fig. 6. Responses of Maglev system with the tuned controllers to periodic output disturbance](image)

As it can be observed in Fig. 6, Fuzzy-PID controller outperforms PID and FOPID controllers in dynamic performance and capability from disturbance rejection point of view.

Additionally, RMS values of error signals for each controller which has been applied to the Maglev system are reviewed, and shown in Fig. 7. By reviewing these values, one can clearly conclude that Fuzzy-PID controller presents enhanced stability and dynamic response, even under disturbance, for the Maglev system.
Another practice to be carried out is to examine the system performance under other disturbance characteristics. For this purpose, a sinusoidal waveform is input as reference signal:

\[ r(t) = 5.8 + 1.8 \sin \frac{\pi}{2} t. \]  

The results are illustrated in Fig. 8. Obviously, the Fuzzy-PID controller has shorter settling time and better trajectory tracking performance as compared to the others. Additionally, the statistical representation of variation of control input values for the trajectory tracking are given in Fig. 9.

A further evaluation and comparison of controllers being studied is the control effort generated by means of the control voltage. In Fig. 9, it is shown that the control effort generated by the Fuzzy-PID controller is considerably less than that of the PID and FOPID controllers. Consequently, it can be stated that the Fuzzy-PID controller provides the best performance with much less control effort.
Furthermore, in order to test the performance of the Maglev system with the PID, FOPID and Fuzzy-PID controllers tuned by CS, the variations of the coil current constant ($k_1$) and the sensor gain constant ($k_2$) are changed from their nominal values in the range of $-20\%$ to $20\%$. The transient and steady state performances of the PID, FOPID and Fuzzy-PID controllers are illustrated in Fig. 10 for $-20\%$ of the nominal amplitude both of the coil current constant $k_1$ and the sensor gain constant $k_2$. From Fig. 10, it can be concluded that the Fuzzy-PID controller tuned by CS outperforms the others in terms of transient and steady state analysis.

The dynamic responses including maximum overshoot, settling time, rise time and steady state error are illustrated in Fig. 11 for $20\%$ of the nominal amplitudes of $k_1$ and $k_2$. It can be seen from Fig. 11, that the CS based Fuzzy-PID controller has
more advantages in comparison to the CS based PID and FOPID controllers under the condition of the parameter uncertainty. Moreover, the objective function values based on the best optimal gain results of each controller in the presence of parameter variation are shown in Fig. 12 by using radar map. It can be concluded from the figure that the optimized Fuzzy-PID controller can provide a lower objective function in comparison to that of the optimized PID and FOPID controllers. As a result, the CS based Fuzzy-PID controller is the best solution considering the dynamic response characteristics and performance index among the CS based PID and FOPID controllers.

Fig. 11. Performances of the controllers for an amplitude of +20 % of \( k_1 \) and \( k_2 \).

Fig. 12. Performance index with different controllers in the presence of parameter variations
6. Conclusion

In this paper, the optimal performance in controlling the Maglev system is sought by tuning three different controllers, conventional PID, Fractional Order PID (FOPID) and Fuzzy PID with Cuckoo Search (CS) algorithm, which is known as the most effective swarm-intelligence based algorithm. An objective function is proposed to evaluate these three different controller’s dynamic response in perspective of performance criteria of overshoot, the rise time, the settling time and the steady state error in step response and trajectory tracking. Reviewing these results, one can clearly conclude that CS-tuned Fuzzy PID outperforms conventional PID and FOPID. Following the evaluation of step response, different input signals such as square wave and sinusoidal, also with added disturbance are investigated, and, CS-tuned Fuzzy PID has demonstrated an outstanding performance and robustness again under these conditions in comparison to conventional PID and FOPID. Moreover, several uncertainties are introduced to monitor the efficiency of controllers, and it was observed that the Fuzzy-PID tuned with CS has achieved outstanding performance compared to PID and FOPID.

References


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