Reducing power losses is invested in the trend of combating climate warming. It is necessary to know what parameters of power transmission lines affect the losses in them. In short and medium lines with accounted lumped parameters, the role and influence of the line parameters on losses are visible. In the lines with distributed parameters, at least with one series parameter and one parallel parameter, the role played by them, computing losses in ordinary way as difference between power at line sending and receiving end, is hidden. This is pronounced when considering parallel lines. In two parallel lines the losses can be greater than in a single line. This may occur when the current at the end of the lines is less than the boundary value: the value when two parallel lines and a single line have the same losses. The longer the line and the higher the rated voltage, the stronger the effect. In view of this aspect, it is necessary to know the boundary current. This current can be determined in ordinary way by a series of calculations changing the load value. In some cases, boundary current is affected not only by modulus of the current at the end of the line but also by its angle. It is better to calculate the boundary current by a formula, while studying the role of each parameter.

**Keywords**: Distributed parameters, long lines, losses of power lines, medium lines, parallel power lines.
1. INTRODUCTION

Today, all the possibilities are relevant, which do not worsen the thermal balance of the globe. Even a small reduction in energy loss is worth considering. Attention paid to losses in power systems is huge [1]–[5].

Climate warming is hampered by the use of renewable energy sources. This requires developed networks and modern power lines. Although high temperature conductors increase losses, they promote the use of renewable energy sources, thereby reducing greenhouse gas emissions [6], [7].

This article focuses on losses. The sophisticated approach to the design and operation of power transmission lines [8] contributes to their more rational use and reduction of losses in the face of increasing use of renewable energy sources. With modern calculating technology, the calculation of losses in the power transmission line is not onerous. You just need to subtract the power at the end of a line from the power at the beginning of the line. The power losses are affected by the parameters of the power line as well. The role of these parameters, with such a calculation, is hardly visible and may be unexpected. The alternative determination (ad) of losses makes it possible to more clearly see this influence. Therefore, it is possible to find the reason for the deviation of losses from the expected, more reasonable and quick to select the design and parameters of the line. Thanks to the advances in semiconductor technology, flexible ac transmission system finds application in the electric power industry [9], [10]. It is desirable to configure line performance influencing its parameters as needed that is easier to do if you know their impact.

An AD approach is useful not only for power losses. It can also be used to calculate voltage losses [11] for the same purpose, to more clearly reveal the influence of line parameters.

Talking about power losses, the AD is useful considering the losses in parallel lines.

Parallel transmission lines are widely used in the electric power industry [12]. This allows, firstly, increasing the reliability of power supply [13], facilitating the scheduling of repairs, as well as increasing the power transmission capacity. This is important when transferring the energy from one power system to another [14]. At the same time, when the wires of both lines are suspended on the same supports, the right of way does not increase [15]. Then we have double-circuit lines (DCL) [16], [17] where a total transposition is made, to eliminate the influence of one line on the other [15], [18].

Thus, energy transfer along two parallel lines (TPL) is used quite often.

When one of the TPL for some reason remains in operation, the active losses on the route change. Whether the second line should be switched on afterwards or not, can also be considered from the point of view of active losses.

Here a criterion is the boundary current (BC): if currently the load current is more than BC, the disconnected line must be switched on. This is because the losses in TPL are less when the current at the end of the lines (further – load current (LC)) is more than BC and vice versa.

The issue of losses in parallel lines with distributed parameters is not specifically considered. For instance, in [4], [19]–[21] the losses are studied but not in parallel lines. The Depazo formula is given in [22] but this expression cannot be used to decide whether two or one line should be in opera-
As said above, for a given load, line loss, be it single line or parallel lines, is defined self-explanatory, as the difference in power at the transmitting end of the line and at the receiving end. A series of calculations on the set of currents at the end of the lines is required, to determine the BC. For identical parallel lines (that have the same specific parameters \( r_0, x_0, g_0, b_0 \) and length \( l \)), it is necessary to vary only the absolute value of the load current (LC). However, sometimes parallel lines can be removed from one another, and in addition have different lengths and parameters [18], in short, they are non-identical. Then it is necessary to determine BC changing the modulus of LC at a specified angle \( \varphi \). With the modern development of digital technology, this also is not a serious obstacle. But finding the BC analytically is in itself attractive and, moreover, the role of each line parameter can be estimated. In addition, solving the problem for non-identical lines allows you to solve the problem in a routine way, when the number of parallel lines is more than two. The final expression of BC for identical lines is simple and easy to calculate, but for non-identical lines it is complicated and rather tortuous to find. To derive this expression, it is necessary to intricate AD of power loss in long lines. The expression for BC of identical lines is found as a special case.

Four specific distributed parameters describe the power line: resistance \( r_0 \), reactance \( x_0 \), conductance \( g_0 \) and susceptance \( b_0 \). Whether distributed or lumped parameters are used in the calculations or altogether are taken into account depends on the length of the line and its rated voltage. The shorter the line and (or) lower the rated voltage, the fewer parameters are considered distributed or taken into account at all.

Low voltage lines are considered as short (<80 km) where parallel parameters – conductance and susceptance – are skipped (considered equal to zero); these lines are characterised by two series parameters – resistance and reactance; such lines can be considered as two parameter lines. In medium voltage line conductance is considered to be zero; only three parameters are taken into account. The parameters of low voltage short (<80 km) and medium voltage (80 – 250 km) lines [3] are considered as lumped in order to simplify calculations, although in the three parameter lines they can be used as distributed depending on the required accuracy of calculations especially in cases where the line length is respectful. Cable lines do not fit this gradation. In long lines, all four parameters are taken into account such as they are in every line by nature, i.e. as distributed. Component lines (CL) of a TPL (especially of DCL) for the most part are identical.

When the lines CLa and CLb are identical, the BC \( \hat{I}_a \) or \( \hat{I}_b \) is the same, no matter which of the two lines is disconnected. When the lines CLa and CLb are non-identical, the BC \( \hat{I}_a \) when CLb could be disconnected is different from BC \( \hat{I}_b \) when CLa could be disconnected.

The BC can be determined if at least one series parameter \( r_0 \) or \( x_0 \) and one parallel parameter \( g_0 \) or \( b_0 \) is taken into account.

In the paper, all electrical quantities are considered for one phase: phase current, phase voltage, power and losses for one phase.

All complex quantities are indicated by dot above (e.g., \( \hat{H} \)), absolute value (modulus) is without dot \( (H) \), the real value is indicated by subscript \( r \) \( (H_r) \), and the imaginary value – by subscript \( i \) \( (H_i) \).

All units are in SI system: m, A, V, W, VA, Var, Ω, Ω/m, S, S/m unless otherwise stated or for euphony.
2. DETERMINATION OF POWER LOSSES IN POWER LINES WITH LUMPED PARAMETERS

This consideration was made a long time ago. For example, it is described in [23]. Here a line is considered that has not only specific susceptance $b_0$ but also conductance $g_0$. With lumped parameters, only short and medium lines are considered, in which conductance is not taken into account [21].

Thus, a simpler T-shaped equivalent circuit is proposed for consideration, which provides a clearer representation of the capacitive currents. To write a simple expression for power loss in such lines, it is assumed that the voltages at the beginning and the end of the line are the same. If such an assumption is unacceptable and an exact solution to the issue is necessary, then digital technology allows solving this issue for medium lines as for long ones, especially since the method with different voltages but with lumped parameters also does not provide an exact solution.

In a line (Fig. 1) LC flows from left to right. The current of lumped capacity in the left half of the line flows from left to right and in the right half from right to left. Both currents flow to the lumped capacity at the middle of the line.

Therefore, active losses $\Delta P$ in resistance of the line can be obtained by simple determination (SD) as below:

$$\Delta P = \left(\frac{1}{2}\right) \left[ (I_l^2 + (I_l + \frac{I_c}{2})^2) + (I_l^2 + (I_l - \frac{I_c}{2})^2) \right] = \left(\frac{r}{2}\right)[2(I_l^2 + I_c^2) + I_c^2/2] = r(I^2 + I_c^2/4). \quad (1)$$

The expression is simple, later it will be determined how accurate the results it gives.

Short parallel lines always have less power loss than a single line at any load and line length.

For medium voltage lines, this cannot be so categorically asserted. Therefore, for them you need to try to find BC. For identical TPL, as mostly is the case in medium parallel lines, this phase current can be estimated using the diagram in Fig. 1 (where only one CL is depicted).

For identical lines, the current is distributed equally and common active loss is

$$\Delta P_A = 2 \left(\frac{r}{2}\right) \left[ \left(\frac{I_l}{2}\right)^2 + \left(\frac{I_l + I_c}{2}\right)^2 \right] + \left[ \left(\frac{I_l}{2}\right)^2 + \left(\frac{I_l + I_c}{2}\right)^2 \right] = r\left(\frac{I^2}{2} + \frac{I_c^2}{2}\right). \quad (2)$$

\[\text{Fig. 1. Lumped power line equivalent circuit (one phase)}\]
Equating both expressions, we find BC by (SD):

\[ I_A = I_C / \sqrt{2}. \] (3)

When the modulus \( I \) of the LC \( \dot{I} \) (irrespective of \( \cos \phi \)) is less than the current \( \dot{I}_X \), a single line in operation will have advantage because an active loss now is less than in two such lines connected in parallel.

The higher the capacitive current \( I_C \), the stronger attention should be paid to the issue.

3. THE CASE OF LONG LINES

Consideration of long lines begins according to the diagram in Fig. 2. The desired quantities of a line (\( \dot{U}, \dot{I} \)) at the beginning of line rote (left) are indicated. The given values of the quantities at the end of the line (\( \dot{U}, \dot{I} \)) are indicated on the right.

\[
\begin{align*}
\dot{U}_1 &= \dot{U}_0 + j\dot{U}_{1l}; \quad \dot{I}_1 = |\dot{I}_1| \\
\ddot{S}_1 &= P + j\dot{Q}; \quad S = |S| \\
\end{align*}
\]

\[ l = I + j\dot{I}; \quad \dot{I} = |\dot{I}| \]

\[ \dot{S} = P + j\dot{Q}; \quad S = |S| \]

Fig. 2. Power line and significant quantities.
All quantities are for one phase.

For consideration, we need the following relations, some of them are known [24]; others are taken from [11] and some others – shown in Fig. 2:

\[ \begin{align*}
\dot{z}_0 &= r_0 + jx_0; \quad \dot{y}_0 = g_0 + jb_0; \quad \dot{z} = \dot{z}_0 l; \quad \dot{y} = \dot{y}_0 l; \quad \dot{Z}_\lambda = \sqrt{\dot{z}_0 \dot{y}_0} = Z_{\lambda r} + jZ_{\lambda i}; \\
Z_\lambda &= |\dot{Z}_\lambda|; \quad \dot{\gamma} = \sqrt{\dot{z} \dot{y}} = \alpha + j\beta; \quad \dot{U}_\lambda = j\dot{Z}_\lambda U_{\lambda r} + jU_{\lambda i}; \quad \dot{U}_\lambda = |\dot{U}_\lambda|; \quad \dot{I}_\lambda = \frac{\dot{U}}{\dot{Z}_\lambda} = I_{\lambda r} + jI_{\lambda i}; \\
l_\lambda &= |\dot{I}_\lambda|; \quad \dot{U}_1 = \dot{A}\dot{U} + \dot{B}\dot{I}; \quad \dot{I}_1 = \dot{C}\dot{U} + \dot{D}\dot{I}; \quad \dot{S} = \dot{U}\dot{I}; \quad \dot{S}_1 = \dot{U}_1\dot{I}_1; \quad \Delta \dot{S} = \dot{S}_1 - \dot{S}; \\
\dot{A} &= \text{ch}(\dot{\gamma}); \quad \dot{B} = \dot{Z}_\lambda \text{sh}(\dot{\gamma}); \quad \dot{C} = \text{sh}(\dot{\gamma}) / \dot{Z}_\lambda; \quad \dot{D} = \dot{A}.
\end{align*} \tag{4}
\]

Further, you should pay attention to avoid mistakes by confusing the vector of a quantity (e.g., \( \dot{H} \)) with modulus \( H \) of this quantity.

Based on formula (20) in [11], active loss in line consists of five terms:

\[ \Delta P = c_1 + c_2 + c_{III} + c_{IV} + C_V, \] (5)

where each term is

\[
\begin{align*}
&c_1 = Z_{\lambda r} \text{sh}(2\alpha)(I_{\lambda r}^2 + I^2)/2; \quad c_2 = Z_{\lambda i} \sin(2\beta)(I_{\lambda i}^2 - I^2)/2; \\
&c_{III} = [\text{sh}^2(\alpha) - \sin^2(\beta)]P; \quad c_{IV} = (Z_{\lambda r}^2 - Z_{\lambda i}^2)[\text{sh}^2(\alpha) + \sin^2(\beta)]P/Z_{\lambda r}^2; \\
\end{align*}
\] (6)
\[ c_V = 2Z_{\alpha r}Z_{\alpha l}[sh^2(\alpha) + \sin^2(\beta)]Q/Z_{\alpha l}^2. \] (7)

If we replace \( P \) with \( UsI \) in \( c_{III} \) and in \( C_{IV} \) and replace \( Q \) with \( U\sqrt{1-s^2}(-w)I \) in \( c_V \) then for load at the end of the lines with characteristics \( s \) and \( w \) (see below) we get:

\[
\begin{align*}
    c_{III} &= [sh^2(\alpha) - \sin^2(\beta)]UsI; \\
    c_{IV} &= (Z_{\alpha r}^2 - Z_{\alpha l}^2)[sh^2(\alpha) + \sin^2(\beta)]UsI/Z_{\alpha l}^2; \\
    c_V &= 2Z_{\alpha r}Z_{\alpha l}[sh^2(\alpha) + \sin^2(\beta)]U\sqrt{1-s^2}(-w)I/Z_{\alpha l}^2; \\
    s &= \cos(\phi), w = \text{sign}(I_i),
\end{align*}
\] (8)

where \( \phi \) is a power factor of the LC (at the right in Fig. 2), \( I_i \) – imaginable (reactive) LC.

We combine \( C_{III} \) with \( C_{IV} \) and denote the sum by \( c_3 \), denote term \( c_V \) by \( c_4 \), then we get

\[
\begin{align*}
    c_3 &= c_{III} + c_{IV} = \{[sh^2(\alpha) - \sin^2(\beta)] + (Z_{\alpha r}^2 - Z_{\alpha l}^2)[sh^2(\alpha) + \sin^2(\beta)]/Z_{\alpha l}^2\}UsI \\
    c_4 &= 2Z_{\alpha r}Z_{\alpha l}[sh^2(\alpha) + \sin^2(\beta)]U\sqrt{1-s^2}(-w)I/Z_{\alpha l}^2.
\end{align*}
\] (10)

Separating coefficients \( a \) from \( c_1 \ldots c_4 \) so that they do not contain \( I_i, I \), we get \( a_1 \ldots a_4 \)

\[
\begin{align*}
    a_1 &= Z_{\alpha r}sh(2\alpha)/2; \\
    a_2 &= Z_{\alpha l} \sin(2\beta)/2; \\
    a_3 &= \{[sh^2(\alpha) - \sin^2(\beta)] + (Z_{\alpha r}^2 - Z_{\alpha l}^2)[sh^2(\alpha) + \sin^2(\beta)]/Z_{\alpha l}^2\}Us; \\
    a_4 &= 2Z_{\alpha r}Z_{\alpha l}[sh^2(\alpha) + \sin^2(\beta)]U\sqrt{1-s^2}(-w)/Z_{\alpha l}^2.
\end{align*}
\] (11)

Now active losses can be determined by intricate (observing (5)-(11)) expressions (ID):

\[
\Delta P = c_1 + c_2 + c_3 + c_4 = a_1(l_\alpha^2 + l^2) + a_2(l_\alpha^2 - l^2) + a_3 I + a_4 I.
\] (12)

Separating in (12) \( l_\alpha^2 \) and \( l^2 \) we rewrite it:

\[
\Delta P = a'I^2 + b'I + c'; a' = a_1 - a_2; b' = a_3 + a_4; c' = h'l_\alpha^2; h' = a_1 + a_2.
\] (13)

Thus, in an AD of power loss in long line, the power loss is defined by: 1) the square of LC modulus, 2) the first degree of the modulus, taking into account the angle \( \phi \) (hidden in the coefficient \( b' \)) between the current and voltage and 3) the term defined only by the parameters of the line; in other words, in long line, the power loss is a tri-

160
nomial [25] of LC modulus with a specific angle \( \phi \) of LC.

From expression (13), we see that a constant term that does not depend on LC participates in the loss of power, which makes it feel itself more at low LCs. To decrease the current \( I_A \), it is necessary to increase \( \dot{Z}_A \) (see (4)), i.e. increase \( z_0 \) or decrease \( y_0 \) (their imaginary components) taking into account dependence of \( h' \) on \( z_0 \) and \( y_0 \).

If the line is designed to transmit high power, then the first term plays the main role in the losses where the hyperbolic sine plays the predominant role and it is proportional here to phase resistance \( r_0 \) (see (34) later) but \( \dot{Z}_A \) changes little with \( r \). The second term has its share as well, where square of hyperbolic sine and sine are decisive.

Other influences can be analysed in the same way. Similarly, an expression for reactive power loss can be derived. The influence of parameters on voltage loss can be analysed based on (13) in [11].

Parallel lines are considered based on Fig. 3. All expressions (4)–(13) are valid for component lines CLa or CLb by adding to the subscript of a quantity letter \( a \) or \( b \). The quantities of TPL working together contain letter \( p \) in subscript.

![Fig. 3. Two parallel lines, any of them can be switched off.](image)

We shall look for such an LC, for example, \( I_A \), at which the loss \( \Delta P_{Aa} \) of CLa single in operation is equal to the loss \( \Delta P_{Ap} \) of TPL, CLa and CLb:

\[
\Delta P_{Aa} = \Delta P_{Ap}
\]

or, in a more general case, equal to the share \( k_A \) of loss \( \Delta P_{Ap} \):

\[
\Delta P_{Aa} = k_A \Delta P_{Ap}.
\]

\[
(a'_a - a'_p k_A) I_A^2 + (b'_a - b'_p k_A) I_A + (c'_a - c'_p k_A) = 0.
\]

Denoting below, we obtain current \( I_A \):

\[
A_A = a'_a - a'_p k_A; \quad B_A = b'_a - b'_p k_A; \quad C_A = c'_a - c'_p k_A;
\]

\[
I_A = \frac{(-B_A + \sqrt{B_A^2 - 4A_A C_A})}{2A_A}.
\]

Without going into deep substantiation, the plus sign in front of the square root can be explained as follows. The coefficient \( A_A \) is always positive, since the square of the load current in two parallel lines will always provide less losses than the square
of the same current in one line. The coefficient \( C_A \) is always negative, since the no-load losses in two lines are always greater than in one. Thus, \( |\sqrt{B_A^2 - 4A_A C_A} | > | -B_A | \).

In the vicinity of \( k_A = 1 \) the current is positive, a plus sign is needed in front of the square root.

If the lines CLa and CLb are non-identical and the line CLb is left in the work, then the indices \( a \) and \( A \) are replaced by \( b \) and \( B \) to denote corresponding quantities for CLb.

To use formula (20), you need to know the coefficients \( a'_a, b'_a, c'_a \), the current \( I_{aA} \) and \( a'_p, b'_p, c'_p, I_{pA} \). The necessary quantities for CLa and CLb can be found observing (4), (11) and (13); the quantities \( A_p, B_p, C_p, Z_{Ap} \) for TPL – out of those of CLa and CLb (see Section V in [11]). After that coefficients \( a_{1p} \ldots a_{4p} \) can be determined using the following equalities [26]:

\[
\begin{align*}
\hat{e} &= s h(\hat{y}_p) = s h(\alpha_p + j\beta_p) = \cos(\beta_p) s h(\alpha_p) + j\sin(\beta_p) c h(\alpha_p); \\
\hat{f} &= c h(\hat{y}_p) = c h(\alpha_p + j\beta_p) = \cos(\beta_p) c h(\alpha_p) + j\sin(\beta_p) s h(\alpha_p). \\
\end{align*}
\]  

(21)

Hyperbolic tangent of argument \( \alpha_p \),

tangent of argument \( \beta_p \) and arguments themselves are:

\[
\begin{align*}
th(\alpha_p) &= \text{real}(\hat{e})/\text{real}(\hat{f});
\text{tang}(\beta_p) &= \text{imag}(\hat{e})/\text{real}(\hat{f});

\alpha_p &= \text{arcth}(\alpha_p); \beta_p = \text{arctang}(\beta_p).
\end{align*}
\]  

(22)

(23)

The coefficients \( a_{1p} \ldots a_{4p} \) for arguments \( \alpha_p \) and \( \beta_p \) are determined on the base of (11).

After determination of BC, we can find out how much the loss in the single CLa (or CLb) differs from the loss in the TPL for a given LC \( \hat{I}_d \). If you are willing to know how only the absolute value \( I_d \) of current \( \hat{I}_d \) is affected, then in (16), (17) the modulus of current \( I_d \) must be inserted instead of the BC \( I_A \).

Such a case may arise, when you want to find discrepancy \( \Delta \Pi_d \) which is the difference of losses of TPL and of single CLa at current \( I_{Ad} \neq I_A \); \( \Delta \Pi_d \) for BC \( I_A \) can be designated \( \Delta \Pi_A \) if \( I_{Ad} < I_A \) and \( \Delta \Pi_A \) if \( I_{Ad} > I_A \):

\[
\begin{align*}
\Delta \Pi_A &= \Delta P_{Apl} - \Delta P_{Aal}; \\
\Delta \Pi_A &= \Delta P_{Aam} - \Delta P_{Apm},
\end{align*}
\]  

where \( \Delta P_l \) is by (16), (17) at the current \( I_{Ad} = I_{Al} = q_I I_A \), \( q_I < 1 \); \( \Delta P_m \) – at the current \( I_{Ad} = I_{Am} = q_m I_A \), \( q_m > 1 \). At boundary current \( I_A = I_{Aa} \) discrepancy is zero and \( q = q_I = q_m = 1 \).

If the losses for the vector \( \hat{I}_d \) of a given LC are of interest, such coefficients \( a_{3ad}, a_{4ad} \) and module \( I_d \) should be used instead of \( a_{3a}, a_{4a} \) in expression (11) for current \( \hat{I}_d \) characterised by \( s_d \) and \( w_d \):
\[ a_{3a} = \left[ (sh^2(\alpha_a) - \sin^2(\beta_a)) + (Z_{\alpha a}^2 - Z_{\delta a}^2) [sh^2(\alpha_a) + \sin^2(\beta_a)] / Z_{\alpha a}^2 \right] U s_d; \]
\[ a_{4a} = 2Z_{\alpha a}Z_{\delta a} [sh^2(\alpha_a) + \sin^2(\beta_a)] U \sqrt{1 - s_d^2 / (-w_d)} / Z_{\alpha a}^2 \]

or
\[ a_{3a} = a_{3a} s_d / s; a_{4a} = a_{4a} \sqrt{1 - s_d^2 / (-w_d)} / \sqrt{1 - s^2} / (-w), \]

where \( s_d \) and \( w_d \) by (9) for the given current \( \dot{I}_d \).

4. THE CASE OF TWO IDENTICAL LONG LINES

To derive the BC formula, it is necessary to represent first two terms in (6) in a different form:
\[ c_{1a} = c_{10a} + a_{1a} I_A^2; c_{2a} = c_{20a} - a_{2a} I_A^2; \] (27)

Then formula (12) for the BC \( I_A \) or \( \dot{I}_B \) in a single CLa (or CLb) takes the form:
\[ \Delta P_{Aa} = c_{10a} + a_{1a} I_A^2 + c_{20a} - a_{2a} I_A^2 + a_{3a} I + a_{4a} I. \] (29)

For identical CLa and CLb the loss will be
\[ \Delta P_A = 2[c_{10a} + a_{1a} I_A^2 + c_{20a} - a_{2a} I_A^2 + a_{3a} I/2 + a_{4a} I/2]. \] (30)

Equating the losses \( \Delta P_{Aa} \) and \( \Delta P_A \) we get:
\[ a_{1a} I_A^2 - a_{2a} I_A^2 = 2(c_{10a} + c_{20a}) \] (31)

and with (28) in mind, BC is determined by short expression
\[ I_A = \sqrt{2} I_{\alpha a} \sqrt{(a_{1a} + a_{2a}) / (a_{1a} - a_{2a})}. \] (32)

With identical lines, BCs are equal, \( I_A = I_B \), and are the same regardless of \( s \) and \( w \), i.e. TPL have a single value of BC depending only on the parameters of the lines.

When calculating losses by (16), (17), it should be remembered that
\[ I_{\alpha} = 2I_{\alpha a}; a_{1p} = a_{1a} / 2; a_{2p} = a_{2a} / 2; a_{3p} = a_{3a}; a_{4p} = a_{4a}. \] (33)

This is derived assuming that for identical lines \( \dot{z}_{0p} = \dot{z}_{0a} / 2, \dot{y}_{0p} = 2\dot{y}_{0a} \). Further observing (4): \( \dot{z}_{\lambda p} = \dot{z}_{\lambda a} / 2, \dot{y}_p = \dot{y}_a \). Hence (see (11)), owing to \( \dot{z}_{\lambda p r} \) and \( \dot{z}_{\lambda p i} \), \( a_{1p} \) and \( a_{2p} \) decrease but \( a_{1p} \) and \( a_{2p} \) do not change because the influence of \( \dot{z}_{\lambda p r} \) and \( \dot{z}_{\lambda p i} \) is compensated by \( Z_{\lambda p}^2 \).
Calculations are made for one phase.

A. 110 kV lines

Let us consider what the values can be in the medium voltage line. We take two identical 110e3 V line l=100e3 m long with phase voltage $=110e3/\sqrt{3}$. With wire AC120/27 [23] specific phase parameters are: resistance $r_0=2.53e-4$ Ω, reactance $x_0=4e-4$ Ω, conductance $g_0=0$ S, susceptance $b_0=2.7e-9$ S.

SD is employed according to (1)–(3). For the entire line: phase conductor resistance $r=r_0l=25.3$; $x=40$; phase susceptance $b$ of the line is $b=b_0l=2.7e-9*100e3=270e-6$. Capacitive phase current is $l_c=bU_{ph}=270e-6-110e3/\sqrt{3}=17.1474$ A. BC by (3) is $I_A=I_c/\sqrt{2}=12.125$. Line active loss by (1) is $\Delta P_{AA}=25.3(12.125^2+17.1474^2/4)=5.5793e3$; active loss $\Delta P_A$ of TPL for $I_A$ according to (2) has the same value.

Let us calculate this loss by AD (16), (17) for the same current 12.125 A. We obtain $\Delta P_{AA}=6.186e3$; $\Delta P_A=6.807e3$. 

We see two deviations from the values of SD by (1) and (2): they are not equal to each other and to those of SD. This is because by AD (20) the BC differs from that calculated by SD (2). The BC by (20) is $I_A=I_B=14.0108$ and (29): $\Delta P_{AA}=\Delta P_{BB}=7.4298e3$; $\Delta P_A=\Delta P_B$ have the same value. For current 12.25, $\Delta P_{AA}$ by SD divided by $\Delta P_{AA}$ by AD is $5.5793e3/6.807e3=0.82$; for current $I_C=17.1474$, the ratio is 0.94; for $2I_C=0.98$.

Usually in practice LC is higher, and the accuracy of active loss calculation by SD is sufficient. Ratio of BCs is $12.125/14.0108=0.874$.

B. 330 kV lines

I. Calculations of five sets of line parameters are made with the main purpose of determining their influence on losses in the line (Table 1). Losses $\Delta \hat{S}$ were calculated in ordinary way by formula (4) and to check the correctness of AD of the active losses – also by formula (13). The influence of specific parameter digressions is made by comparison with the basic case. The role of the parameters is visible if we take into account their ratios at their practical values in the electric power systems. In this case based on (25) – (28) in [11], we have the following approximate relations:

$$\psi \approx -r_0/x_0; \quad \gamma_0 \approx \beta_0 \approx \sqrt{b_0x_0}; \quad \alpha_0 \approx \beta_0 \psi/2 \approx r_0 \sqrt{b_0/x_0}/2; \quad (34)$$

$$Z_A \approx \sqrt{x_0/b_0}; \quad U_A = I_\lambda \sqrt{x_0/b_0}; \quad I_A = U \sqrt{b_0/x_0}.$$  

A decrease in susceptance $b_0$ was expected to result in greater diminishing in losses in the absence of load current (at $I=0$) because $I_\lambda$ decreased. This did not happen due to the coefficients $a_1, a_2$ (see (11)) and their dependence on $b_0$ through $\beta_0$ and $\alpha_0$. 


We see observing (34) that $\beta_0$, $\alpha_0$ and $I_0$ diminish proportionally to $\sqrt{B_0}$. Effect of $I_0$ is reduced by increasing effect of $h'$ (see (13)).

The greatest influence on losses is exerted by the resistance $r_0$, since fading decrement $\alpha$ is proportional to $r_0$ but $a_1$, $a_2$, $a_4$ are proportional to $sh(2\alpha)$, $sh(\alpha)$ which are proportional to small values of $\alpha$. At no load, $r_0$ decrease loses its power.

### Table 1. Impact of line parameters of specific quantities

<table>
<thead>
<tr>
<th>Option of parameters</th>
<th>$\tilde{Z}_\lambda$</th>
<th>$I_\lambda$</th>
<th>$\check{\gamma}$</th>
<th>Power line losses $\Delta S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic, see under tab.; further are the regressions from basic</td>
<td>$a_1$</td>
<td>$a_2$</td>
<td>$h'$</td>
<td>$a_3$</td>
</tr>
<tr>
<td>$b_0=5e-10$</td>
<td>0.046e2-2.204e1</td>
<td>236.5</td>
<td>0.0163+0.1202i</td>
<td>$2.528e5-5.393e6i$</td>
</tr>
<tr>
<td>$a'=18.1042$</td>
<td>13.101-5.0032</td>
<td>8.0978</td>
<td>85.813</td>
<td>0</td>
</tr>
<tr>
<td>$\tau_0=1e-5$</td>
<td>3.024e2+9.239e0</td>
<td>630</td>
<td>0.0068+0.3175i</td>
<td>$4.744e5-3.561e7i$</td>
</tr>
<tr>
<td>$a'=2.9386$</td>
<td>2.067e0-0.871e6</td>
<td>1.1954</td>
<td>14.291</td>
<td>0</td>
</tr>
<tr>
<td>$x_0=1e-4$</td>
<td>1.762e2+4.806e11</td>
<td>1042.6</td>
<td>0.0526+0.1845i</td>
<td>$6.86e5-3.732e7i$</td>
</tr>
<tr>
<td>$a'=17.9522$</td>
<td>9.2991-8.6681</td>
<td>0.631</td>
<td>96.779</td>
<td>0</td>
</tr>
<tr>
<td>$g_0=0$</td>
<td>3.037e2-2.846e11</td>
<td>624.6</td>
<td>0.0299+0.3189i</td>
<td>$2.373e5-3.561e7i$</td>
</tr>
<tr>
<td>$a'=17.5526$</td>
<td>9.080e0-8.472e2</td>
<td>0.6082</td>
<td>11.379</td>
<td>0</td>
</tr>
</tbody>
</table>

Basic o.: ph. wire 2AC240/32; $r_0=6.05e-5$; $x_0=3.2e-4$; $g_0=4.132e-11$; $b_0=3.5e-9$; $l=300$ km

Parameter deviations are chosen artificially; $a_4=0$ because $s=0$, $b'=a_3$.

A decrease in reactance $x_0$ leads to a decrease of $Z_\lambda$ and to an increase of $I_\lambda$; $a_1$, $a_2$ change little, therefore, $\Delta P$ at $l=0$ little changes. For the same reason $\Delta P$ at $l=300$ hardly changes. If we are talking about losses, then $x_0$ has little effect.

Decreasing the conductance $g_0$; impendence $y_0$ does not change considerably, since the main role is played by $b_0$. As a result $\tilde{Z}_\lambda$ and $\check{\gamma}$ do not change much, which results that modules of $a_1$ and $a_2$ are even closer than in a basic case, so the effect is felt by $h'$ and less felt by $a'$; as a result $\Delta P$ at $l=0$ diminishes considerably and $\Delta P$ at $l=300$ – only slightly.

II. For some LC the losses of TPL or single CLa or CLb were calculated. Phase wire of CLa is 2AC240/32: $r_{oa}=6.05e-5$; $x_{oa}=3.2e-4$; $g_{oa}=4.132e-11$; $b_{oa}=3.5e-9$; $l_{oa}=300$ km; phase wire of CLb is 2AC300/39: $r_{ob}=4.9e-5$; $x_{ob}=3.2e-4$; $g_{ob}=2.984e-11$; $b_{ob}=3.5e-9$; $l_{ob}=200$ km.

The calculation was carried out for 9 options of initial conditions: 1) $s=1$; $q=1$; 2) $s=1$; $q=0.7$; 3) $s=1$; $q=1.3$; 4) $s=0.8$; $w=1$; $q=1$; 5) $s=0.8$; $w=1$; $q=0.7$; 6) $s=0.8$; $w=1$; $q=1.3$; 7) $s=0.8$; $w=1$; $q=0.7$; 9) $s=0.8$; $w=1$; $q=1.3$. Options 1) – 3) are made with active LC ($s=1$); options 4) – 6) – with inductive LC ($w=-1$), options 7) – 9) – with capacitive LC ($w=1$). Losses $\Delta S$ were calculated in ordinary way to show real and imaginary component. The real component was checked by AD.

In order not to abuse numbers, Table 2 shows options 1), 2), 3), 4). $I_\lambda$ is BC when in operation is TPL or single CLa. $I_{Ad}=I_{a}$ when $q=1$; $I_{Ad}=I_{Al}$ = 0.7$I_{A}$; $I_{Ad}=I_{Am}$ = 1.3$I_{A}$; $\Delta S_{Ap}$ is power loss of TPL at LC $I_{A}$; $\Delta S_{Aa}$ – of single CLa at LC $I_{A}$; $\Delta \Pi_{Al} = \Delta P_{Ap} - \Delta P_{Aa}$.
is discrepancy of active losses of TPL and single CLa at a current \( I_{A} = I_{Al} \); \( \Delta \Pi_{Am} = \Delta P_{Aa} - \Delta P_{Ap} \) is discrepancy of active losses of single CLa and TPL at current \( I_{Ad} = I_{Am} \). The CLa with more resistance than CLb takes less BC. When \( w=1 \), the CLa takes still less BC than CLb.

Table 2. Results of calculating currents and power losses in TPL and single line.

<table>
<thead>
<tr>
<th></th>
<th>( s=1; q=1 )</th>
<th>( s=1; q_{1}=0.7 )</th>
<th>( s=1; q_{m}=1.3 )</th>
<th>( s=0.8; w=1; q=1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_{A} )</td>
<td>141.3</td>
<td>141.3</td>
<td>141.3</td>
<td>180.9</td>
</tr>
<tr>
<td>( I_{Ad} )</td>
<td>141.3</td>
<td>98.91</td>
<td>183.7</td>
<td>180.9</td>
</tr>
<tr>
<td>( \Delta S_{Ap} )</td>
<td>1.037e6-6.032e7i</td>
<td>9.711e5-6.061e7i</td>
<td>1.126e6-5.997e7i</td>
<td>9.047e5-6.258e7i</td>
</tr>
<tr>
<td>( \Delta S_{Ad} )</td>
<td>1.037e6-3.421e7i</td>
<td>8.541e5-3.506e7i</td>
<td>1.284e6-3.319e7i</td>
<td>9.047e5-3.718e7i</td>
</tr>
<tr>
<td>( \Delta \Pi_{Al} )</td>
<td>3.725e8-0</td>
<td>1.17e5</td>
<td>1.576e8</td>
<td>0</td>
</tr>
<tr>
<td>( \Delta \Pi_{Am} )</td>
<td>3.725e8-0</td>
<td>4.47e5</td>
<td>1.576e5</td>
<td>0</td>
</tr>
<tr>
<td>( I_{B} )</td>
<td>437.7</td>
<td>437.7</td>
<td>437.7</td>
<td>374.9</td>
</tr>
<tr>
<td>( I_{Bd} )</td>
<td>437.7</td>
<td>306.4</td>
<td>569</td>
<td>374.9</td>
</tr>
<tr>
<td>( \Delta S_{Bp} )</td>
<td>2.133e6-5.46e7i</td>
<td>1.511e6-5.793e7i</td>
<td>2.972e6-5.001e7i</td>
<td>1.363e6-6.184e7i</td>
</tr>
<tr>
<td>( \Delta S_{Bd} )</td>
<td>2.133e6-1.324e7i</td>
<td>1.186e6-1.92e7i</td>
<td>3.413e6-5.248e6i</td>
<td>1.363e6-2.008e7i</td>
</tr>
<tr>
<td>( \Delta \Pi_{Bl} )</td>
<td>-2.98e-8</td>
<td>3.255e5</td>
<td>-4.16e5</td>
<td>-1.49e-8</td>
</tr>
<tr>
<td>( \Delta \Pi_{Bm} )</td>
<td>2.98e-8</td>
<td>-3.255e5</td>
<td>4.16e5</td>
<td>1.49e-8</td>
</tr>
</tbody>
</table>

Identical lines: \( r_{0}=6.05e-5; x_{0}=3.2e-4; g_{a}=8.264e-11; b_{0}=3.5e-9; l=300 \text{ km} \)

With active LCs (\( s=1 \)) BCs are \( I_{A}=141.3, I_{B}=437.4 \). When LC contains inductive current (\( s=0.8, w=1 \)) then \( I_{A}=180.9, I_{B}=374.9 \). With the same modulus but capacitive LC (\( s=0.8, w=1 \)) – \( I_{A}=111.1, I_{B}=509.1 \).

Let us recall that losses at BC are losses when \( q=1 \) i.e. real(\( \Delta S_{Ap} \)) = real(\( \Delta S_{Ad} \)) = \( \Delta P_{A} \). Observing Table 2 and the rest five calculated cases, the ratio between discrepancies \( \Delta \Pi \) and the losses \( \Delta P \) at BC is obtained: with an active LC (\( s=1 \)) \( \Delta \Pi_{Al}/\Delta P_{A}=0.11; \Delta \Pi_{Am}/\Delta P_{A}=0.15; \Delta \Pi_{Bl}/\Delta P_{B}=0.15; \Delta \Pi_{Bm}/\Delta P_{B}=0.21 \); with inductive LC (\( s=0.8, w=1 \)) \( \Delta \Pi_{Al}/\Delta P_{A}=0.16; \Delta \Pi_{Am}/\Delta P_{A}=0.24; \Delta \Pi_{Bl}/\Delta P_{B}=0.21; \Delta \Pi_{Bm}/\Delta P_{B}=0.27 \); with capacitive LC (\( s=0.8, w=1 \)) \( \Delta \Pi_{Al}/\Delta P_{A}=0.09; \Delta \Pi_{Am}/\Delta P_{A}=0.11; \Delta \Pi_{Bl}/\Delta P_{B}=0.12; \Delta \Pi_{Bm}/\Delta P_{B}=0.17 \).

For identical lines with CLa parameters those figures are: \( \Delta \Pi_{Al}/\Delta P_{A}=0.17; \Delta \Pi_{Am}/\Delta P_{A}=0.23 \). The discrepancy is more with the greater LC deviation from BC: for \( q_{1}=0.5 \), \( \Delta \Pi_{Al}/\Delta P_{A}=0.25 \); for \( q_{m}=1.5 \), \( \Delta \Pi_{Am}/\Delta P_{A}=0.41 \); for \( q_{m}=2 \), \( \Delta \Pi_{Am}/\Delta P_{A}=0.99 \).

With significant LC deviations from BC, it is necessary to pay attention to the losses.

In cable lines, these issues manifest themselves with less voltages and distances.
6. CONCLUSIONS

1. Active power losses in transmission lines, with at least one series \((r_0, x_0)\) and one parallel \((g_0, b_0)\), distributed parameter taken into account, can be alternatively determined as trinomial of the modulus of the load current at the end of the line with the given angle \(\varphi\) between voltage and current. Losses consist of three terms: one term is proportional to the square modulus, the second term is proportional to the modulus and dependent on angle \(\varphi\) and the third term is independent of the load current. The proportionality coefficients and the third term are functions of the line parameters.

These lines operating in parallel can be characterised by a boundary current such that one line operating at a load below boundary current has less losses than two lines operating in parallel at the same load and vice versa. If two lines are identical, the formula of boundary current is short and not dependent on angle \(\varphi\).

2. An alternative determination of losses allows you to identify the influence of each parameter and to derive the expression for boundary current.

3. Specific resistance \(r_0\) has the greatest influence on losses since fading decrement \(\alpha\) is almost proportional to it; specific reactance \(x_0\) has almost no effect; conductance \(g_0\) affects mainly at low load currents; the decrease in susceptance \(b_0\) does not lead to a strong decrease in losses, since other quantities are influenced here.

4. In medium lines, a simple determination of active losses already at a load current over twice the capacitive line current has an error under -2 %. For normal line load currents, the accuracy of simple loss determination is sufficient. By simple determination, the boundary current of identical lines over the entire range of medium line lengths is \(\approx 86\%\) of the true value.

5. Losses from inadequate operation of parallel lines can reach a significant amount compared to minimal losses.

ACKNOWLEDGEMENTS

The research has been supported by “Future-Proof Development of the Latvian Power System in an integrated Europe (FutureProof)”, project No. VPP-EM-INFRA-2018/1-0005.

REFERENCES


