EFFICIENT POSITIONING OF TWO LONG-RANGE PASSENGER AIRCRAFT IN FORMATION FLIGHT

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Abstract
In today’s world, each airline is forced to look for new savings opportunities. One of the methods may be the use of formation flights in daily flight operations, which may allow a reduction in fuel consumption by several percentages. The paper presents the genesis of how the consideration of such flights and the possibility of their implementation in an airline had started. The leader’s plane generates vortices, which, with the proper alignment of the planes to one another, can reduce the drag on the wingman. However, the wrong position may not only have no positive effect but also may be a threat to stable wingman flight. The article presents a method of using these vortices in such a way as to have a positive impact on the aerodynamics of the wingman. A favourable position in the vertical and horizontal axes will be determined in relation to the vortex generated by the leader’s plane in order to obtain the greatest benefit in reducing fuel consumption. The paper presents an operational analysis of the possibility of maintaining such a distance to obtain profit on fuel but also to ensure the highest level of safety of the flight.

Keywords: formation flight; optimisation; strip theory; dynamics of flight
Type of the work: research article

1. INTRODUCTION

Rising costs of fuel, human labour and aviation parts could negatively affect the quality of services offered by airlines and ticket prices. However, all air operators are looking for savings in other aspects of their business. In order to reduce expenses on maintenance and spare parts, they are replacing the aircraft fleet with a younger one. The connection network is optimised to minimise the time that the plane remains on the ground. Specialised software is purchased to accelerate the work of handling agents or airline dispatchers.

Similarly to travelling by car, the vehicle must be refuelled before driving. For the Boeing 767, which uses 60 tons of fuel in one flight [1], with the price per ton around US $800, this amounts to around US $48,000. In the case of LOT Polish Airlines (PLL LOT), the replacement of the fleet from the Boeing 767 to the 787 type allowed a savings of 20% of fuel on one flight [2].

The International Civil Aviation Organization (ICAO) recommends all operators to create their own fuel-saving policy [3]. One of the recommendations is, for instance, to wash the airframe regularly. The deposited dirt increases the drag force and, therefore, the thrust required by the engines for flight. Each airline is looking for its own unique solutions that allow it to be one step ahead of the competition all the time.

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Useful solutions should also be sought in the animal world. Observations and analyses of bird formation flights resulted in undertaking research aimed at the application of a given solution in the aviation industry. Over the past decades, many scientific articles have been written showing the positive impact on reducing fuel consumption during a formation flight.

The paper begins with an overview of literature about formation flying. Next, an optimisation model and calculation of longitudinal separation between aircraft in formation are presented. Later, we present a mathematical model for calculation of the best position for the wingman. Then, we use the strip theory to calculate aerodynamic loads. Finally, the conclusions on usage of formation flights are given.

2. LITERATURE REVIEW

2.1. Look at nature

Research [4] carried out in 1970 on birds flying in V-formation key flight birds showed that individuals in a formation can fly 71% longer than in the case of a single bird. Such an effect is possible due to the influence of the circulation generated by the leader on the wingman. This causes changes in both the induced drag and the lift force. Observing various species of birds, it was concluded that the distance in the longitudinal axis of the location of the birds in the formation depends on their wingspan, and it is usually 1/4 of the bird’s wingspan. It was also found that the number of individuals in each arm is not significant.

In 2013, a detailed flight analysis was carried out on the flight of ibises [5]. For this purpose, global positioning system (GPS)-based equipment and inertial measurement unit were used. Devices with a total weight of 23 g were placed on 14 birds. From the 43-min recording, a 7-min fragment was selected and analysed. It was observed that the birds flew in a perfect V-shape, at an angle of 45° to each other. It was calculated that the aerodynamic drag of the wingman was 65% lower than that of the leader. There was a decrease in the flight speed of the entire formation and a decrease in the heart rate of birds by 10%. The above example proves that the energy demand of individuals flying in a formation is reduced. An important observation was that the key leader changed at a constant interval. It was replaced by the one that had not yet been in this position. This allows for uniform tiredness of all individuals flying in the formation.

2.2. Research works

In a previous work [6], an analysis of the safety of planes flying in formation was performed. For this purpose, a computational diagram of the formation of vortices created by the leader and the effect of these vortices on the wing aircraft was presented. Aircrafts with a wingspan of 22 m and 80 m were compared. It was found that, due to the inability to counteract turbulence, a small aircraft cannot be placed behind a large one. It was indicated that when planning formation flights in passenger aviation, it would be necessary to create air corridors depending on the span of the aircraft.

As part of the research on formation flights, it was found that the optimal place to perform a formation flight is the space over the Atlantic Ocean [7]. It has also been determined that the number of takeoffs and landings of flights at the same airports is too low to consider formation flights for them. Therefore, a transatlantic flight simulation was carried out for two aircraft taking off and landing from other airports. The study covered flights that would fly in formation over the ocean. Maximum takeoff weight and cruising speed were assumed for the calculations: >Mach 0.8. From the calculations, it was found that the most optimal combination of aircraft types is the Airbus A380-800 as the leader and the Boeing 747-400 as the wingman, and the fuel gain would be >6%.
2.3. Boeing and FedEx

Boeing, together with the transport company FedEx, analysed the possibility of using formation flights in their network of connections [8]. Optimisation of the creation of aircraft pairs at evening ferryings between bases in the United States was performed. The calculations showed that the fuel savings would be 12.46%. If operated 5 days a week on such a route, this would save $2.8 million. In the next step, the companies considered a flight of two Boeing 777s in a formation flight [9]. In the cruise mode, they achieved 5% fuel savings. In order to carry out this flight, additional onboard equipment was installed on the wingman. For using today’s passenger planes in formation flights, it is important that one of the devices, namely, traffic collision avoidance system (TCAS), undergo a software update. This shows that the currently installed devices on airframes do not need to be replaced with new ones, but only little changes in their software must be made.

3. OPTIMISATION

3.1. General

In order to determine the optimal position on the vertical and lateral axes of the wing aircraft in relation to the leader’s aircraft so as to obtain the greatest benefit from the vortex of the flight, an optimisation process was performed. This action is aimed at determining the best (optimal) solution from the point of view of a specific criterion (or mathematically as a search for the extreme of the assumed function) [10]. The best result obtained is called ‘optimal’. The design process begins with identifying the problem to be solved. For this purpose, the basic decision variables of the issue and their limitations are described. Then, the objective function is created, the complexity of which depends on the number of parameters assumed. The next step in the optimisation process is the selection of the appropriate optimisation procedure, a tool that will be used to solve a given problem. Each question will require different methods to obtain an effective answer.

One of the computational forms of the optimisation problem is the steepest descent method. It consists of looking for the minimum of the function in the direction opposite to that of the gradient of the function at the starting point. To determine the next iterative step, it is necessary to determine the gradient of the function at the starting point and the coefficient of the step. The process of the steepest descent method is based on an algorithm, which — in every step in the predefined direction — is looking for the minimum value of the objective function. The search is finished when it is impossible to obtain a smaller value than for step $i - 1$. The disadvantage of the steepest descent method is the slow convergence in the case of a badly chosen point of departure.

MATLAB (1 Apple Hill Drive, Natick, MA 01760-2098) software was used to solve the optimisation process. Different types of optimisation methods, such as linear minimisation, nonlinear minimisation, multicriterion optimisation, or least-squares optimisation, can be used.

3.2. Longitudinal separation

Determining an appropriate longitudinal separation between planes participating in a formation flight cannot be treated only as a question about optimising engine thrust savings. It is necessary to verify the resulting aerodynamic loads, turbulence and, above all, the safety aspect. In the following section, an analysis is made to determine the optimal location based on the collected data on the behaviour of the vortex, published information from flight tests and the human factor in the aspect of maintaining safety. Aviation authorities have defined the minimum longitudinal separation between airplanes in order
to maintain the highest possible level of flight safety [11]. For formation flights, it is necessary to build an accurate model of the vortex and be able to analyse it during this type of flight. To this end, the team at the German Aerospace Centre analysed the structure of the resulting vortex and defined the following ranges of the vortex, counting the distances from the leader’s aircraft [12]:

- 1.5–15 of the airplane span: the range of the strongest influence of the vortex but with the smallest possibility of describing its structure;
- 15–150 of the airplane span: the range of moderate vortex influence, but of a stable structure;
- ≥150: the range of the vortex dispersion.

The above ranges are given in terms of distances calculated as the product of the airplane span, because the size of the vortex depends on the length of the wings of the leader’s airplane. As part of the research, analytical models for calculating the parameters of the vortex were also developed. The calculations obtained for the exemplary vortices were very similar to the results determined experimentally.

When analysing the position of the wing aircraft in relation to the leader, the vortex descent aspect should be taken into account. For this purpose, an analysis of the descent speed in relation to the flight speed of the aircraft generating a given vortex was performed [13,14]. Accordingly, it was defined that the rate of descent is 7 ft/s, or 2.1 m/s.

For an airplane with a wingspan of about 50 m and a limiting value of 15 m, the longitudinal separation is 750 m. At the cruising altitude, this leg is about 3 s and the vortex descends by about 6 m. Airplanes used for transatlantic flights are usually at least 10 m high. In changing situations, such as gusts of wind or engine disturbances, 3 s is a very short time for the pilot of the wing aircraft to complete any manoeuvre. A slight reduction in speed by the leader or an increase by the wingman could cause the crash of both planes. The characteristics of the vortex, i.e., the fact that it descends, can potentially ensure the safety of the flights. With vertical separation equal to the height of a wing plane, changes in position in the longitudinal axis would not increase the danger. Accordingly, the span limit of 15 m cannot be complied with. The larger one should be taken so that the difference in height is at least 10 m. According to the calculation, the vortex will descend by this value in about 5 s, which is a distance of about 1,300 m.

The minimum separation determined here is close to the values assumed for the performed flight tests. In a test conducted by Boeing [9], a longitudinal separation of 1,200 m was established. The same distance was assumed for a National Aeronautics and Space Administration (NASA) flight on Gulfstream C20A planes [15]. In both tests, this value was assumed due to the previously mentioned aspect of the vortex structure. In terms of its stable structure, this distance allows the greatest use of its operating force.

4. CIRCULATION

4.1. Mathematical model

In a plane, during a flight, the pressure is lower than the surrounding atmospheric pressure on the upper wing surface and higher on the bottom wing surface. As a result, the air flowing around the upper and bottom surfaces tends to flow from the inner part of the wing to the wing tip, symmetrically on both sides of the wing. Merging of both streams creates vortices, one per wing tip and both of the same size. The measurable value of the vorticity is the operator called $\Gamma$ circulation. It is a measure of the intensity of the vortex. The dependency between circulation and lift was described by the Russian scientist Nikolay Zhukovsky in 1905 [16]. It defines that the lift depends only on the size of the vortex circulation, the flow velocity and the density of the medium in the circular profile.
where $m$ – leader weight, $g$ – standard gravity, $\rho$ – air density, $b_0$ – the span correction adopted due to the elliptical distribution of the circulation on the wing and $V$ – velocity.

Having the value of circulation generated by the flying plane, we can then use the Biot–Savart law to determine the induced velocity at any point $P(x,y,z)$ away from the vortex \[17,18\].

\[ \Delta q = \frac{\Gamma}{4\pi} \frac{dl \times (r_0 - r_1)}{|r_0 - r_1|^3}, \]  

where $\Delta q$ – value of the induced speed at the point $P$, $dl$ – vorticity component and $r_0 - r_1$ is the distance of the point from the vortex, calculated as the sum of the vectors from the centre of the coordinate system.

After several transformations enabling the calculation of the induced velocity along the three axes $u$, $v$ and $w$, the following equations were obtained:

\[ u = K (r_1 \times r_2)_x \]  
\[ v = K (r_1 \times r_2)_y \]  
\[ w = K (r_1 \times r_2)_z \]

where

\[ K = \frac{\Gamma}{4\pi \|r_1 \times r_2\|^2} \left( \frac{r_0 \cdot r_1}{r_1} - \frac{r_0 \cdot r_2}{r_2} \right) \]

\[ \Gamma = \frac{m^* g}{\rho^* b_0^* V}, \]  

Figure 1. Distribution of forces, including the induced velocity, where: $L$ – lift, $D$ – aerodynamic drag, $V$ – horizontal velocity, $W$ – vertical velocity, $\alpha$ – angle of attack.
The effect of the vertical component of the induced velocity is the generation of an additional pitch of the aircraft with respect to the original angle of attack (Figure 1). This additional angle is called the induced angle, calculated from the following operation:

$$\alpha_i = \frac{w}{v}$$  \hspace{1cm} (7)

where \( w \) – vertical component of the induced velocity.

At the moment of the aircraft’s pitch, an additional force is generated from the lifting force acting in the longitudinal direction of the aircraft. It is equal in value to the induced drag generated by the vortex operating on the wing.

$$L' = L \cdot \tan \alpha_i$$  \hspace{1cm} (8)

In a fixed flight, the thrust of the engines balances the aircraft’s drag. The generated component of the lift force increases the value of the force pushing the aircraft. However, when viewed from the opposite side, the value of the induced drag decreases the drag.

$$T = D - L'.$$  \hspace{1cm} (9)

4.2. Adopted assumptions

The calculations were based on the values for the Boeing 767 [19] airplane, flying at cruising altitude. The selected parameter values are as follows:

- \( b = 47.24 \) m – wingspan;
- \( S = 283.35 \) \( \text{m}^2 \) – wing surface area;
- \( V = 265 \) \( \text{m/s} \) – velocity;
- \( m = 130,000 \) \( \text{kg} \) – weight; and
- \( \rho = 0.36 \) \( \text{kg/m}^3 \) – air density at flight level (FL)-380.

The centre of the coordinate system was assumed in the centre of the airfoil. However, the point \( P(x,y,z) \) has been defined, from the leader side, as the point at the follower’s wing tip. Points \( 1(x_1,y_1,z_1) \) and \( 2(x_2,y_2,z_2) \) indicate the extreme points of the leader’s wing (Figure 2).

Figure 2. The adopted coordinate system.
4.3. Results

The results of position optimisation on the $y$- (lateral) and $z$- (vertical) axes for the distance from the leader on the $x$-axis determined in the previous section is presented. The function to be minimised is the value of the thrust required to fly. Graphs of the necessary thrust for the flight are presented (Figure 3, 4). Three different starting points were used for optimisation, defining the initial position of the wing tip in relation to the leader:

$$(y, z)_1 = (-b, -b); (y, z)_2 = (0, 0); (y, z)_3 = (b, b).$$

The necessary thrust for an undisturbed flight is $T_0 = 54.08$ kN.

![Figure 3. Thrust $T$ at $x = 1,300$ m.](image1)

![Figure 4. Thrust $T$ (2D) at $x = 1,300$ m.](image2)
Table 1. Optimisation results for $x = 1,300$ m.

<table>
<thead>
<tr>
<th></th>
<th>$y = -b, z = -b$</th>
<th>$y = 0, z = 0$</th>
<th>$y = b, z = b$</th>
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<tr>
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<td>1</td>
<td>8</td>
</tr>
<tr>
<td>$y$ [m]</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$z$ [m]</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$w$ [m/s]</td>
<td>$-0.7433$</td>
<td>$-0.7433$</td>
<td>$-0.7433$</td>
</tr>
<tr>
<td>$L$ [N]</td>
<td>4,500</td>
<td>4,500</td>
<td>4,500</td>
</tr>
<tr>
<td>$T$ [kN]</td>
<td>49.5</td>
<td>49.5</td>
<td>49.5</td>
</tr>
</tbody>
</table>

5. STRIP METHOD

5.1. Mathematical model

In order to determine the aerodynamic loads arising on a wingman, the strip method was used, which made it possible to take into account the velocity induced by the vortex. The vortex generated by the leader’s aircraft affects the roll, pitch and yaw ($P$, $Q$, $R$) angular velocities of a winged aircraft.

The sequence of calculations of the strip method were as follows [20,21]:

- The wings were divided into a number of elements (strips).
- The local angle of attack and slip angle, as well as the value of the resultant velocity vector, was determined in each strip. The calculation takes into account the velocity induced by the leader and the effect of the vortices generated by adjacent strips.
- From the aerodynamic characteristics of the profile, the local aerodynamic coefficients of lift and drag are determined.

The total effect of angular velocity on the wing load is represented as the difference between the total load on all velocity components and the load on linear speed.

The final form of the local change of forces in the aircraft system is presented as follows:

$$
\begin{bmatrix}
X_{i}^x \\
Y_{i}^y \\
Z_{i}^z 
\end{bmatrix} = \frac{1}{2} \rho * S * V_{i}^2 * \begin{bmatrix}
-\cos \alpha_i \cos \beta_i & -\cos \alpha_i \sin \beta_i & \sin \beta_i \\
-\sin \beta_i & \cos \beta_i & 0 \\
-\sin \alpha_i \cos \beta_i & -\sin \alpha_i \sin \beta_i & -\cos \alpha_i
\end{bmatrix} \begin{bmatrix}
C_{x_i} (\alpha_i, \beta_i, M_{ai}) \\
C_{y_i} (\alpha_i, \beta_i, M_{ai}) \\
C_{z_i} (\alpha_i, \beta_i, M_{ai})
\end{bmatrix} + \\
-\frac{1}{2} \rho * S * (V_{i}^2)^2 * \begin{bmatrix}
-\cos \alpha_i \cos \beta_i & -\cos \alpha_i \sin \beta_i & \sin \beta_i \\
-\sin \beta_i & \cos \beta_i & 0 \\
-\sin \alpha_i \cos \beta_i & -\sin \alpha_i \sin \beta_i & -\cos \alpha_i
\end{bmatrix} \begin{bmatrix}
-\cos \alpha_i \cos \beta_i & -\cos \alpha_i \sin \beta_i & \sin \beta_i \\
-\sin \beta_i & \cos \beta_i & 0 \\
-\sin \alpha_i \cos \beta_i & -\sin \alpha_i \sin \beta_i & -\cos \alpha_i
\end{bmatrix} \begin{bmatrix}
C_{x_i} (\alpha_i, \beta_i, M_{ai}) \\
C_{y_i} (\alpha_i, \beta_i, M_{ai}) \\
C_{z_i} (\alpha_i, \beta_i, M_{ai})
\end{bmatrix}
$$

where $C_{x_i}$ – dimensionless coefficient of the resistance force for the $i$-th cross section of the wing, $C_{y_i}$ – dimensionless coefficient of the side force for the $i$-th cross section of the wing, $C_{z_i}$ – dimensionless...
coefficient of the lift force for the $i$-th cross section of the wing, $M_{ai}$ – Mach number for the resulting velocity of the flight in the $i$-th cross section of the wing, $M_{ai}^V$ – Mach number for the resulting velocity of the flight in the $i$-th cross section of the wing, $V_{0i}$ – resulting velocity of the flight in the $i$-th cross section of the wing, $V_{0i}^V$ – resulting velocity of the flight in the $i$-th cross section of the wing.

5.2. Adopted assumptions

The calculations were based on the same values and assumptions as in the Section 4.2 for the Boeing 767, flying at cruising altitude: longitudinal distance of planes in the formation: 1,300 m; horizontal and vertical separation of planes in the formation: 0 m. The result of the above assumptions is the occurrence of only the vertical component of the induced velocity by the leader’s plane in relation to the wingman.

- $P = 0.063$ (rad/s) – roll angular velocity.

One half of the wing was divided into 40 equal strips. Calculations were made for the wing.

5.3. Results

The obtained results of the aerodynamic loads resulting from the disturbance caused by the angular velocity on the wing aircraft were compared with the normal ones operating in flight. In order to determine them, the Schrenk method was used [22].

![Figure 5. X-component of the load on the wings.](image1)

![Figure 6. Y-component of the load on the wings.](image2)
6. CONCLUSIONS

For the determined longitudinal separation of 1,300 m, the possible fuel savings are 3.2%. Optimisation for three different starting points showed that such a value is obtained for zero vertical and lateral separation (Table 1). The algorithm for calculating the aerodynamic loads on a wing aircraft made it possible to obtain results in a very short time. Such a code could be implemented in currently available passenger aircraft. Any system modification should not significantly burden the work of the onboard computers, and it is necessary that the data on the effect of the calculations be generated on an ongoing basis, and not with a delay. The obtained values of the aerodynamic loads on the wing of the plane resulting from the angular velocity and velocity induced (Figure 5, 6, 7) were compared with the loads acting in a normal flight. In order to present them on a common graph, due to the large difference in units, those obtained by the strip method had to be multiplied 100 times (the values displayed in Fig. 8 are already increased values). Otherwise, they would not be visible at all. To present them, it was also necessary to change the sign caused by the opposite sense of the adopted coordinate system. The determined values of the loads allow us to state that the disturbances resulting from the formation flight do not adversely affect the structure of the wingman.
REFERENCES